WAVES
of matter

ACTIVITY 10
Wave Functions for Lonely Electrons

> ### *Goal*
> We continue to refine the wave function we use to describe an electron. We will discover that the electron's wave properties actually prevent us from making exact measurements of its position and momentum simultaneously. This result leads to philosophical issues related to the nature of knowledge.

In our descriptions of electrons so far, we have been assuming that electrons move in the beam and all of them have identical energies. This situation has been useful, but it is somewhat artificial. For a real world we need to describe individual electrons as well as beams.

First, let's see why our present version has problems when dealing with an individual electron. Suppose we wish to use a wave function that travels straight across this line of the page. It moves from left to right without changing its speeds. It interacts with nothing. So, the potential energy is zero. One electron, no interactions, and no energy changes. Sounds like a simple situation.

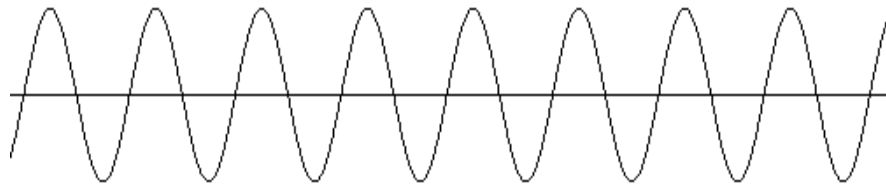The *Wave Function Sketcher* would give us the result in Figure 10-1.

Figure 10-1: The Wave Function Sketcher result for the situation described above.

?   Does this wave function match the description above? Why or why not? (If you have difficulty with this question, think about the probability of finding the electron at various locations.)

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The difficulty is that the probability density is rather uniform across the page. Also, it does not change with time. Let's create a better wave function for a single electron. In the diagram below, sketch an approximate graph of probability density versus position for an electron in the middle of the page. The approximate shape of the wave function, not the numerical value at any point, is the important feature.
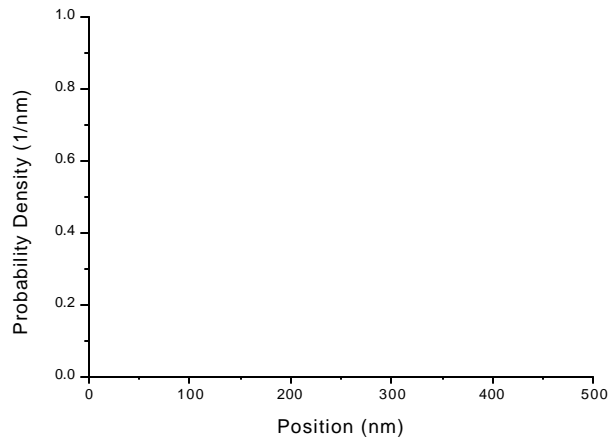
? Explain why you drew the wave function as you did both *near* the location of the electron, and far *from* it.

? How would you expect this wave function to change as the electron moved across the page?

A possible representation for the electron moving across the screen is shown in Figure 10-2. (*Quantum Motion* can show animations of this.)
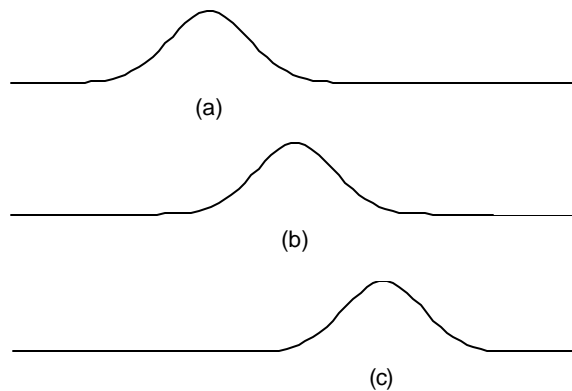

Figure 10-2: Three snapshots of a wave function representing an electron traveling across the page.

This wave function has a probability different from zero only in a small region. It moves as the electron moves. So, it fits the need for our purposes.

All measurements indicate that the electron is a very small object. It is essentially a point in space. So, the "bumps" on the wave functions in Figure 10-2 indicate that the probability of finding the electron is different from zero at several different locations.

Unfortunately, such a wave function does not just "pop out" as a solution to Schrödinger's Equation. So we must learn to build it from other solutions.

The ability of waves to interfere both constructively and destructively enables us to combine several different waves to create *waveforms* with useful shapes. Because waves interfere with one another both constructively and destructively at the same time but in different locations, we can never create a wave function that is not zero at one location, but zero everywhere else. So, to represent a single electron, we must construct a wave function that, when squared, gives a probability density with one maximum, decreases sharply away from that maximum, and then is zero at relatively large distances for where we expect the electron.

Even simple waveforms like the disturbance on the rope can be constructed by adding simple waves together. Our goal will be to combine simple waves to produce a wave function that, when squared, gives a smooth probability density graph similar to Figure 10-3:
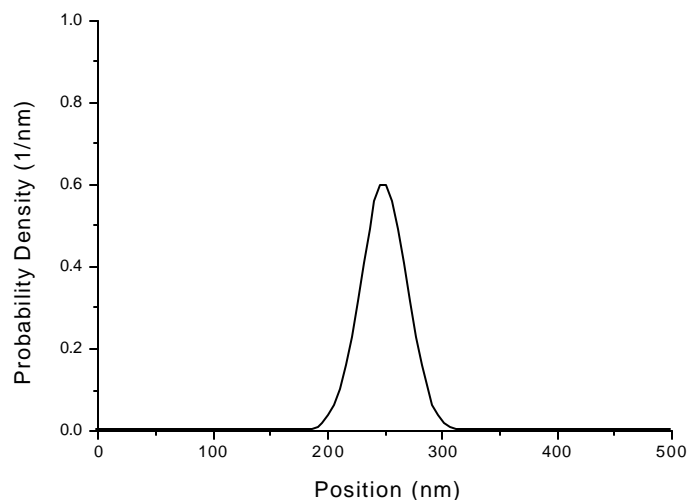

Figure 10-3: Typical shape for the probability density of a *localized* electron.

To see how we might create such a wave look at the addition of waves shown in Figure 10-4.
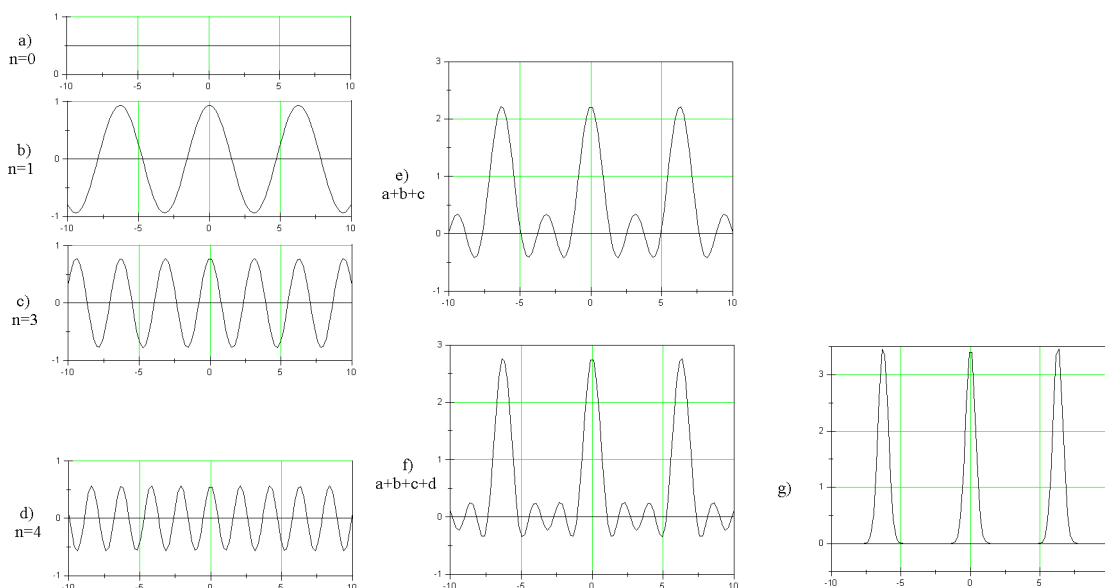
Figure 10-4: the three waves (a-c) on the left, gives the one on the right (e). Adding even more waves can result in a wave form similar to Figure 10-3.

By adding hundreds of waves with carefully selected wavelengths and amplitudes we can come very close to the form in Figure 10-3.

From the de Broglie relation we know that wavelength is related to momentum. So, when we add wave functions of different wavelengths, we are adding wave functions, which represent objects with different momenta.

To create wave functions like this use the *Wave Packet Explorer*. This program allows us to add wave functions of different momenta (wavelengths) rather easily.

Click in the upper left window (*Amplitude vs. Momentum* graph).
A vertical line appears representing a wave function whose amplitude is proportional to the length of the line, and whose momentum is the value at the line on the horizontal axis. The wavelength is calculated from using the de Broglie relation. The wave function appears directly below.

Repeat this procedure by first clicking on the graph in the upper left window, at any value of position and amplitude and creating a new wave function. Each time, a simple wave function is added to all the wave functions that were already present. The individual waves are shown on the graph on the bottom left while the sum of these waves is just above these.

? How does the resulting added wave function change as you increase the number of simple wave functions?

Now, instead of adding just a few wave functions, add a large number of them. Click many times in a definite pattern to create a large number of wave functions. Sketch the resulting wave functions below.

Compare your resulting wave functions with others in the class. Find a couple that are quite different from yours. Describe how their momentum-amplitude graph is different from yours.

Some of these patterns are probably similar to Figure 10-3. Try an amplitude-momentum distribution similar to the one in Figure 10-5.

To come even closer you can add a very large number of momenta. Hold down the shift key and the left mouse button and drag the cursor across the amplitude-momentum graph. This process adds together all of the wave functions with momenta and amplitudes in the shaded region.
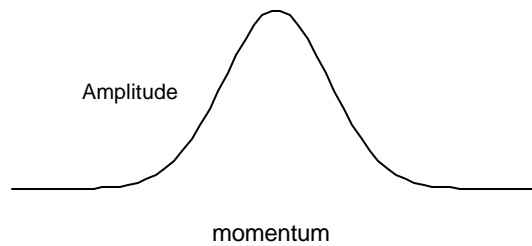
Amplitude

momentum

Figure 10-5: An amplitude-momentum distribution that gives a wave function similar to the one in Figure 10-3.

(You can look at any one wave function in all of the collection by moving the mouse over the line in the momentum graph. Both the momentum line and the corresponding wave function in the bottom graph will turn green.

Sketch the position wave function below.

?   What is the probability interpretation of the wave function?

?   Does this wave function have a large or small range of momenta associated with it?

Now try a wave function which has a very narrow range of momenta. Create it by dragging over a small range.

Sketch the position wave function below.

?   How is the probability interpretation different from the previous wave function?

Now, try to create

■   a wave function that represents a high probability of the electron being in a very small region of space, and

■   a wave function that represents an electron that has equal probabilities of being in many different regions.

?   How do these wave functions (and the momenta represented by them) differ?

We can create a wave function that represents an electron in a small region of space. To do so we must add together many different simple waves. Each simple wave has a different wavelength. Because wavelength is related to momentum, each simple wave has a different momentum.

Thus, we need many different momenta to create a wave function for one electron. We interpret this result to mean that the electron may have any one of the momenta. So, instead of having just one momentum, these localized electrons have a probability of having many different momenta. The probability of each momentum is related to the amplitude of the wave function with that momentum (Figure 10-6).
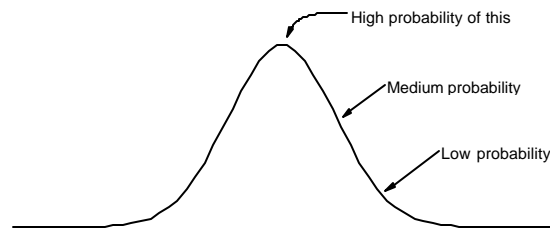

Figure 10-6: The probability of each momentum depends on the amplitude on the amplitude-momentum graph.

We must make a similar statement about position. Even when we add lots of waves with lots of different momenta, we still get a wave function similar to Figure 10-7.
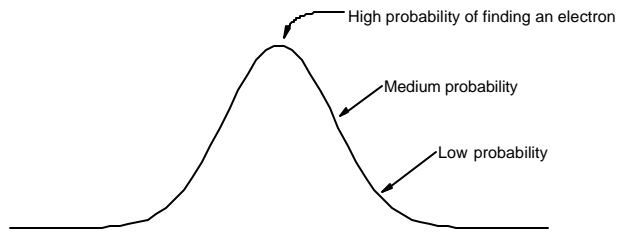
High probability of finding an electron

Medium probability

Low probability

Figure 10-7: The relative probability of finding the electron at various locations for a localized wave function.

Summary
We can create a wave function that localizes the electron to a small region of space. However, not just to one point in space. In doing this we increase the possible range of momenta that the electron can have. We must talk about probabilities for both the electron's location and its momentum.