Name Partners Date

Visual Quantum Mechanics
The Next Generation

Bound States and Binding Energies

Goal

- To build a mental image of bound states using classical systems
- To gain an understanding that bound states are a wave characteristic.

Introduction

When an electron is trapped in a small region, it has unique behaviors that can only be explained with quantum theory. For example, the wave behavior of an electron allows it to exist only in certain states. In this tutorial, we will study what makes a constrained electron have discrete energy levels.

A. Standing Waves

Consider the following experiment: a string with infinite length is attached to an oscillator at one end (see Figure 1 below).

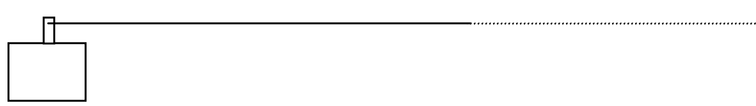

Figure 1: Diagram of an infinitely long string attached to an oscillator.

A-1. When the oscillator is turned on, describe what will happen to the string. Be sure to sketch the shape of the string and explain how you arrived at your conclusions.

Now suppose the string is cut to a length of L tied to a wall (see Figure 2 below).

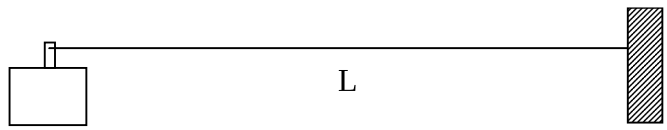

Figure 2: Diagram of a finite length of string attached between an oscillator and a wall.

A-2. When the oscillator is turned on, what will happen to the string? Sketch the possible shapes of the string and explain how you arrived at your conclusions.

A-3. Is it possible to obtain a stable wave pattern on the string? If your answer is "no", explain your reasoning. If your answer is "yes", explain how to do it.

A-4. Using the equipment as illustrated in Figure 2, increase the frequency of the oscillator from 0 Hz to 100 Hz. Describe how the shape of the string changes as the driving frequency increases.

A-5. Can you observe stable patterns? If your answer is "yes", sketch all of the patterns and label them with the conditions with which they are generated.

A-6. What are the similarities and differences for all of the patterns you observed?

A-7. Instead of changing the frequency, are there any other ways that you can produce different patterns? Discuss whether or not the following features will change the patterns and how they might change the patterns. Explain your reasoning.

- the amplitude of the oscillator

- a heavier string

- a tighter string (increased tension in the string)

- a different length of the string

A-8. Now set the frequency of the oscillator to 30 Hz. Try changing the amplitude, tension, and length. Describe how the changes of these features affect the patterns.

A-9. Suppose we have a physical system such that the type of string, the length of the string, and the tension of the string all have fixed values. The only thing we can change is the frequency of the oscillator. As we have discovered by changing the frequency, we can create a series of different stable patterns. Discuss the relationship between the energy of the oscillating string and the frequency of the oscillator when the amplitude of the oscillator is also fixed.

A-10. For the system to have stable patterns, what must be true about the energy? Explain.

B. Bound Electrons

When an electron is trapped in an atom, it is analogous to the case of an electron constrained in a small region. Energy is needed to free the electron from its constrained region. Therefore we can use an energy diagram to represent this system (see Figure 3). The amount of energy needed to free an electron is called the binding energy, E_B.

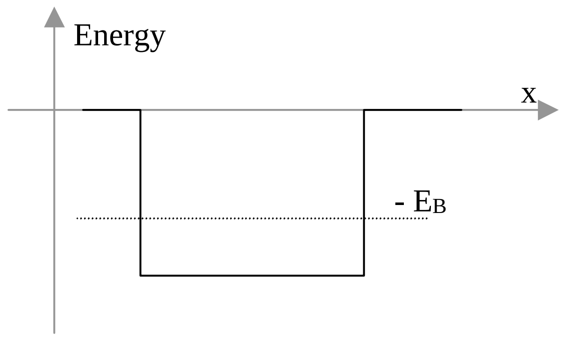

Figure 3: Example of an Energy Diagram.

B-1. Suppose an electron is trapped in a region of space. We describe such a case as a bound state. How much energy do you need to add to move the electron from a bound state with binding energy $-E_B$ to a state at rest outside of the well? Explain your answer.

B-2. Speculate on the factors that you need to consider in order to determine values for the binding energy. State why you think each of them is important.

C. What Energies Are Allowed?

As we have discussed before, electrons behave as waves. We describe such a wave with a quantum wave function. When trapped in a small region, the wave of an electron can form different patterns analogous to the waves on a string.

In this section, we will use a computer program, *Bound States*, to help us find the appropriate wave functions and see how these waves are related to energies for a trapped electron.

C-1. How will the pattern of the wave look for the lowest energy of an electron trapped in the potential energy shown in Figure 3? Use what you have learned from the example with strings to make your prediction.

Now open *Bound States*.

Check your prediction from above by having the computer test it. Click in the potential at a low energy.

C-2. Sketch the wave in the space below.

Choose SOLVE from the menu and try the slightly smaller binding energy, E_B=28.

C-3. Discuss whether or not it is an acceptable wave pattern by making comparisons between the shape on the screen and your observations in the standing wave experiment.

Now try a slightly different binding energy.

C-4. Compare this solution to the previous one. How does it differ?

C-5. How is it similar?

C-6. What characteristic must your wave function satisfy in order to be physically possible? Explain your answer.

C-7. Click on several values for the binding energy. Attempt to narrow the possible value of the energy to be between two numbers. Record those values below and explain why the acceptable energy must be between them.

C-8. Use the arrow up and down keys to make small adjustments in the energy. Get the best possible wave function. How is this wave function similar to the wave on a string?

C-9. How is it different?

Now explore the other wave functions. Search for Allowed Energies under the Energy pull-down menu. By moving the cursor to each energy, you can display the wave function. For each case two wave functions are displayed, but only one of them is acceptable. You need to decide which one.

D-1. Compare the solutions and energies. What general conclusions, if any, can you make about the shape of the wave function?

D-2. Now create more attractive potential by changing the depth to about 500 eV. (Potential pull down, Change depth) How many allowed energies does the computer find?

D-3. Before looking at the wave functions, sketch the approximate shape of each wave function.

D-4. Move the cursor to the fourth state. Is the shape approximately what you expected? If not, discuss the situation with your instructor.

D-5. This example shows that when an electron is trapped in a small region, the wave behavior requires that the electron exist only in certain states. Summarize why and extend the argument to explain why electrons in atoms can have only certain allowed energies.