Exploring Quantum Tunneling

Goal
In this activity, you will investigate the wave function for a tunneling electron and the parameters upon which tunneling depend.

Quantum tunneling is a unique result which can be explained only in terms of the wave nature of matter. Recall that the wavelength of the particle's wave function is inversely proportional to the momentum of the electron. Thus, when the particle's wavelength is large, its momentum is small. For electrons only,

$$\lambda = \frac{h}{p} \qquad (1)$$

Further, recall that the square of the amplitude of the wave in a region is proportional to the probability of finding an electron in that region.

To begin this activity we will explore the properties of the wave function for an electron in a simple potential energy situation. Start the *Quantum Tunneling* program. Go to *File/Open* in the top pull-down menu and open the file *Step.txt*. The top half of the program screen displays a potential energy diagram similar to the one in Figure 1. This potential energy diagram is very similar to the one for an electron in a TV picture tube as it approaches the TV screen.

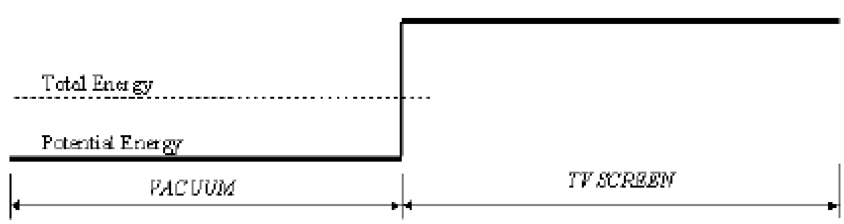

Figure 1: Potential Energy diagram for an electron that moves toward a TV screen

 Visual Quantum Mechanics is supported by the National Science Foundation under grants ESI 945782 and DUE 965288. Opinions expressed are those of the authors and not necessarily of the Foundation.

Click the blinking *Redraw Graphs* button. After calculating for a few seconds, the program will display the wave function in the window just below the potential energy diagram.

Sketch the wave function in the two regions (vacuum and screen).

Look at your sketch above of the wave function. How does the probability of finding the electron in a vacuum compare with the probability of finding it at various locations in the screen?

This program was written for students with very little mathematical skills. So, we simplified the mathematics by defining the "decrease length." This length, L, is the distance where the amplitude of the wave function decreases to a value of 1/e of its value at the boundary as shown in Figure 2.

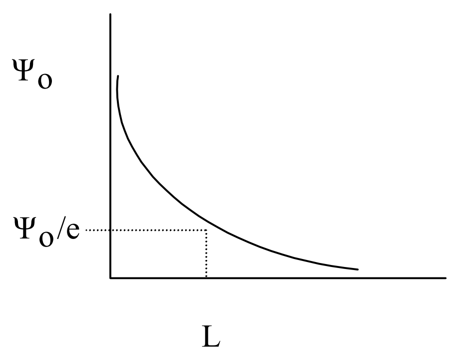

Figure 2: Definition of decrease length.

? Indicate the approximate decrease length on the potential energy diagram shown above.

? If the total energy of the electron increases, predict how the decrease length will change. Explain your prediction.

Thus, the probability of finding the electron in the metal increases as its energy increases.

Now, consider electrons that approach a very thin metal foil (Figure 3). You will investigate the wave function for electrons in this situation.

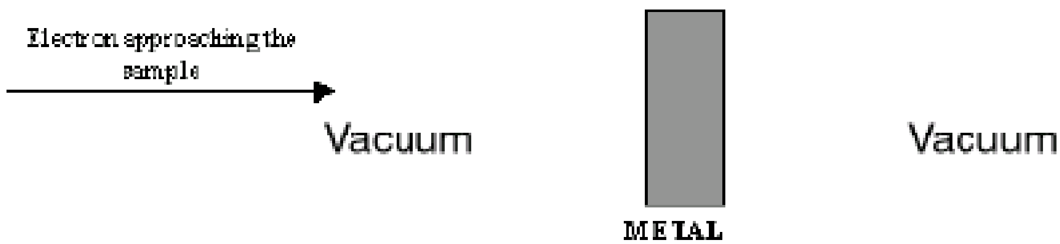

Figure 3: An electron approaching a thin metal foil.

Establish the boundaries where the potential energy changes. Then, draw the potential energy diagram of the electron indicating its potential and total energies inside the metal as well as in the vacuum on either side.

Use the *Quantum Tunneling* program to draw the wave functions for the electrons in each region. Sketch the wave function below.

As you can see the wave at the right edge of the foil is not zero. Because the wave function must be smooth at the boundary and the total energy is greater than the potential energy, an oscillating wave function occurs on the far right. Sketch the probability density for the electron in all three regions.

This result shows that the electron can be on the right side of the potential barrier. Yet, its energy is not large enough to get there. Electrons can sometimes be on this side of metal only because it behaves as a wave. Conservation of energy says that none of them should be there. This effect of electrons being where they should not be according to classical physics is called quantum tunneling.

Quantum tunneling allows electrons and other small objects to have a small probability of being in regions where conservation of energy prohibits them.

So far you have been considering the situation of electrons striking a thin foil. The probability that quantum tunneling occurs depends on the thickness of the material involved. To see this effect increase the width of the potential without changing the energies.

Create the new wave function and sketch it below.

? How has the probability that quantum tunneling will occur changed? Explain your answer.

To explore the dependence of the probability for quantum tunneling on other variables, continue to use the *Quantum Tunneling* program and the investigation of the width of the potential. Try several different widths. Then describe below how the probability of tunneling depends on potential width.

? Now, keep everything fixed except the maximum value (height) of the potential energy. How does the probability of quantum tunneling change as the height of the potential energy changes?

Explain your conclusion in terms of the decrease length.

Repeat the investigation by changing only the total energy of the electron. How does the probability of quantum tunneling change as the total energy changes?

Explain your conclusion in terms of the equation for decrease length.

Summarize all of these results by describing all of the changes that would cause the quantum tunneling to increase. First, describe the result in terms of the graphics you have seen here.

Second, look in your textbook at the mathematical expressions for the wave function in a situation involving a barrier potential, V_o, and a total energy E for which $E < V_o$. The transmission coefficient , T, is approximately

$$T \approx 16 \frac{E}{V_0}\left(1-\frac{E}{V_0}\right)e^{-2\alpha a} \qquad (2)$$

where a is the width of the barrier and

$$\alpha = \frac{\sqrt{2m\,(V_0 - E)}}{\hbar} \qquad (3)$$

Equation (2) is approximately correct when $\alpha a >> 1$.

Describe how the mathematical solutions are consistent with the graphical representations.

Even in situations where a small object does not have sufficient energy, it has a small probability of being detected where energy considerations say it should not be. This phenomenon, called *quantum tunneling*, can occur only because of the wave behavior of electrons. The mathematical treatment of quantum tunneling is presented in your text.