Can Problem Solving in Physics Facilitate Conceptual Change in Mathematics?

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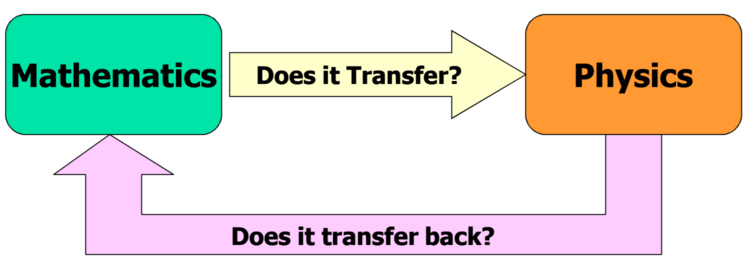
Question Usually Asked…

How does knowledge in mathematics transfer to problem solving in physics?



Question NOT Usually Asked…

Can problem solving in physics affect learning in mathematics?

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Theoretical Framework

- 'Concept Projection'
  - Combination of resources & strategies
  - Used to identify & apply a concept
  - Depends upon the context
- Conceptual understanding involves…
  - several concept projections,
  - each supported by a many shared resources

(diSessa & Wagner, 2005; Wagner, 2010)

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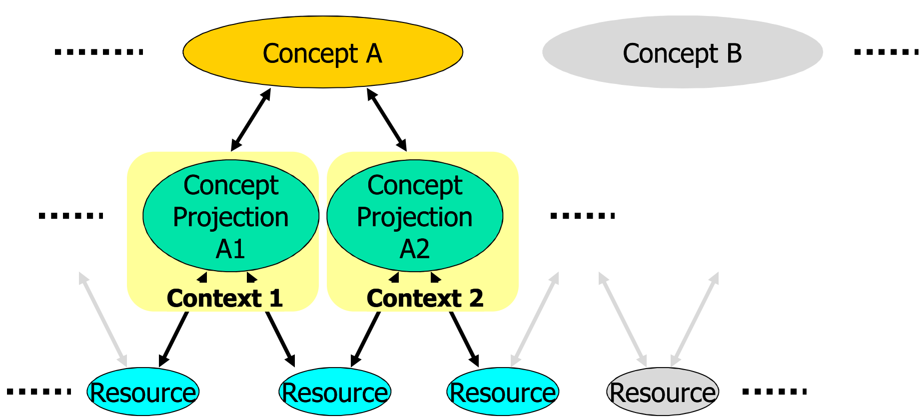
Theoretical Framework



(diSessa & Wagner, 2005; Wagner, 2010)

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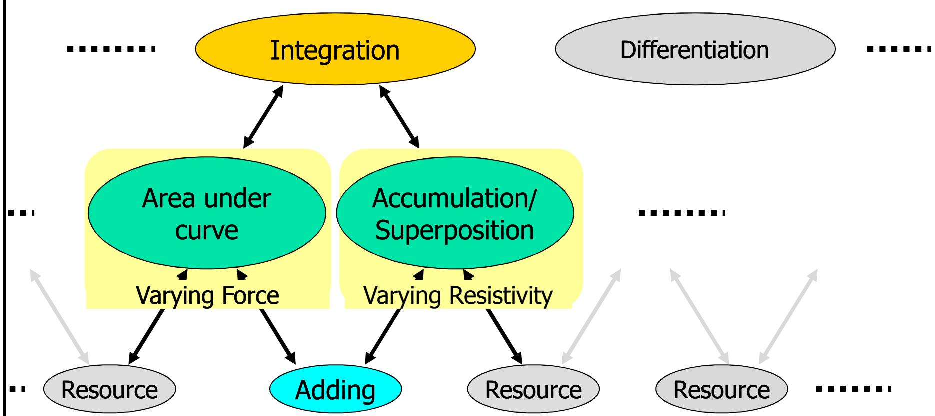
Theoretical Framework: Example



(diSessa & Wagner, 2005; Wagner, 2010)

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Research Question

To what extent are students able to develop concept projections of integration spanning …

- 'area under curve' and
- 'accumulation/superposition'

as they progress through two semesters of a calculus-based physics course?

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Data Collection

- Teaching Interviews[1]
  - Students solved problems
    - Numerical → Graphical → Equational
  - Hints provided to facilitate students' problem solving

Spring 2009 — 1st Semester Calc-Based Physics

n=20 Engin. majors

1 2 3 4

Fall 2009 — 2nd Semester Calc-Based Physics

n=15 Engin. majors

1 2 3 4

(e.g. Hershkowitz, Schwarz, & Dreyfus, 2001)

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Type of Analysis

- Case Study of a 'typical' student – *Sam.*
- Transcript analyzed over several interviews.
- Focus on integration in various contexts.

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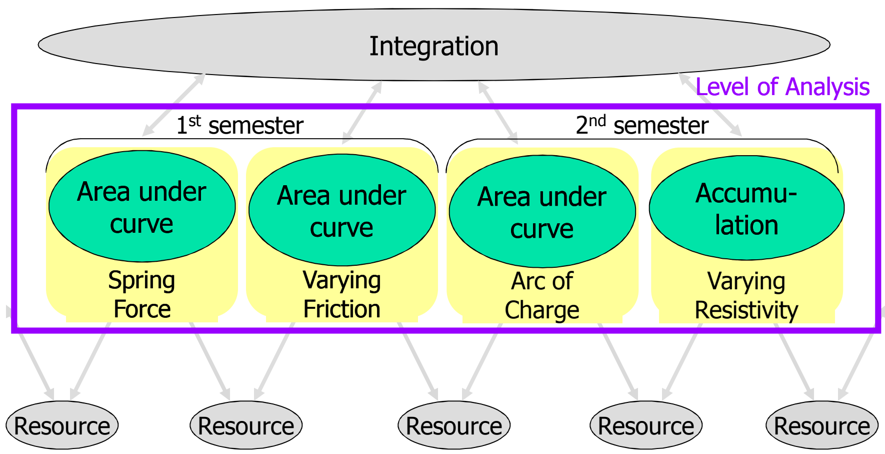
Level of Analysis



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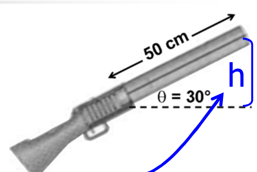
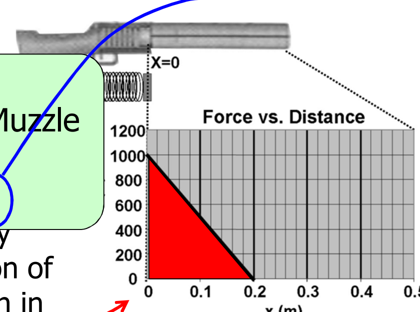
Context: Spring Force

- 1st Semester : Interview 2



A 0.1 kg bullet is loaded into a gun (muzzle length 0.5 m) compressing a spring as shown. The gun is then tilted at an angle of 30° and fired



Correct Solution
Work by Spring = Total Energy at Muzzle
Area under graph = KE + PE
= $\frac{1}{2} mv^2 + mgh$

bullet by the spring as it leaves the fully compressed position varies as a function of its position x (m) in the barrel as shown in the graph below.

What is the muzzle velocity of the bullet as it leaves the gun, when the gun is fired at the 30° angle as shown above?

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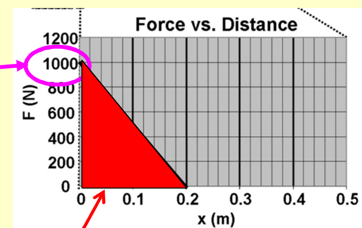
Context: Spring Force

- 1^{st} Semester : Interview 2
  - *Sam* found 'k' from slope & 'x' from intercept; used ½ kx^2 to get work
  - Interviewer probed student further…

I: Can you think of a way to find work done without knowing 'k 'and 'x'?
S: I don't know.
I: What information can you extract from this graph of F vs. x?
S: Spring constant … I don't know … maybe the work.
I: How can you calculate work from this graph?
S: It's force times distance.
I: What value of force do you use?
S: I assume 1000 N.
I: Is the force 1000 N all the way?
S: … No.
I: So what should you do?
S: You have to integrate it through.
I: What does that integration mean on this graph?
S: It's the area under the curve thing.



Context: Varying Friction

- 1^{st} Semester : Interview 4

A sphere radius r = 1 cm and mass m = 2 kg is rolling at an initial speed v_i
o
a
s

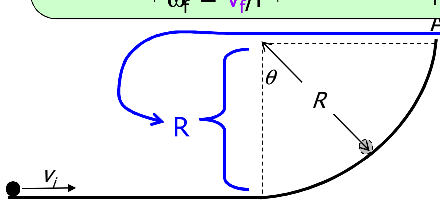
Correct Solution

Final KE − Initial KE = Work Done

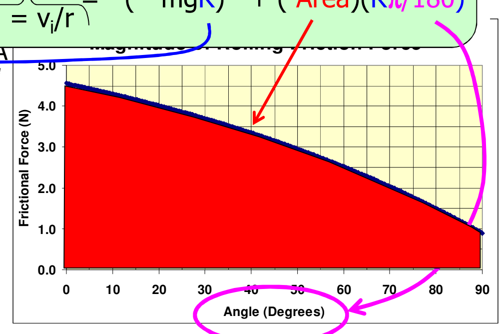
$(\frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2) - (\frac{1}{2}mv_i^2 + \frac{1}{2}I\omega_i^2) = W_{gravity} + W_{Friction}$

$\omega_f = v_f/r$ $\omega_i = v_i/r$

$= (-mgR) + (-\text{Area})(R\pi/180)$



What is the launch speed of the sphere as it leaves point A?



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Context: Varying Friction

- 1st Semester : Interview 4
  - *Sam* had solved a previous problem without friction.
  - Interviewer asked about this problem…

I: How is this problem similar or different from the previous problem?

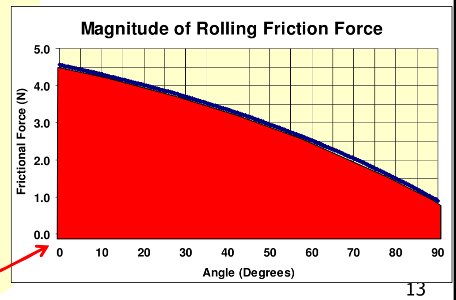
S: Same principles applied. All the values are the same except … you are given a graph to find the work done by friction.

I: How do you find that?

S: … I need to figure out how to do the integral.

I: How do you figure that?

S: … I will need to find the area under this graph.



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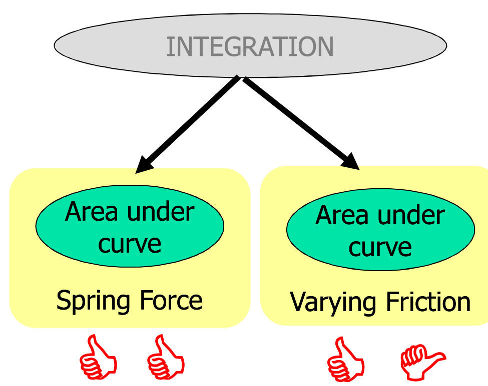
Summary: 1st Semester

- At first, recognize integral as 'area under graph', with some prompting.
- Later, able to use this concept projection in other contexts.
  - Scaffolding needed to facilitate handling subtlety with scaling.



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Correct Solution

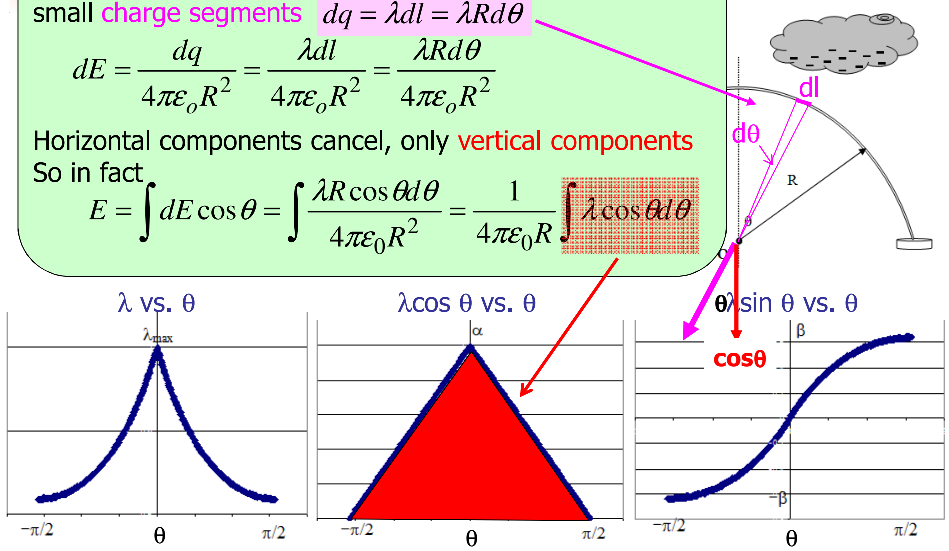
Electric Field = superposition of electric field due to small charge segments $dq = \lambda dl = \lambda R d\theta$

$$dE = \frac{dq}{4\pi\varepsilon_o R^2} = \frac{\lambda dl}{4\pi\varepsilon_o R^2} = \frac{\lambda R d\theta}{4\pi\varepsilon_o R^2}$$

Horizontal components cancel, only vertical components

So in fact

$$E = \int dE\cos\theta = \int \frac{\lambda R\cos\theta d\theta}{4\pi\varepsilon_0 R^2} = \frac{1}{4\pi\varepsilon_0 R}\int \lambda\cos\theta d\theta$$



Find the magnitude and direction of the electric field at your feet (i.e. at point O on the ground directly below the top of the arch).

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Context: Arc of Charge

- 2nd Semester : Interview 1
  - Based on previous problems, *Sam* set up:
  - Interviewer probed further…

$$E = \frac{1}{4\pi\varepsilon_0 R}\int_{-\pi/2}^{\pi/2} \lambda\cos\theta d\theta$$

ral.

I: You have several graphs…which do you choose to find the area under?

S: You better do it under the λcosθ versus θ.

I: How do you know you should use that graph?

S: Just assuming that because we're using cosθ in the integral.

I: Do you mean that because you see the cosine in the integral so you should use the graph concerning cosθ?

S: That's what I thought.

λcos θ vs. θ

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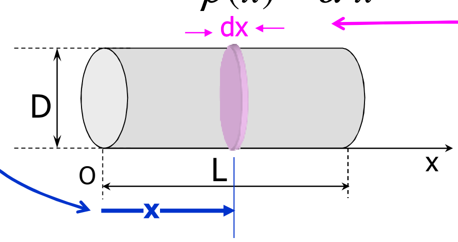
Correct Solution

Total Resistance = Superposition of Resistance dR provided by thin slices of the resistor of length dx

Slice which is distance x from left end provides $dR = \frac{\rho(x)dx}{\pi D^2/4} = \frac{\alpha x dx}{\pi D^2/4}$

Total Resistance = $R = \int dR = \int \frac{\alpha x dx}{\pi D^2/4}$

$$\rho(x) = \alpha\, x$$



where x is the distance from the left end of the conductor

Context: Changing Resistivity

- 2nd Semester : Interview 2

$R = \frac{\rho L}{\pi \frac{D^2}{4}}$

I: So what is the resistance of each slice whose length is dx?

S: ... Do I use this equation [writes equation] with the new ρ plugged in? But your length wouldn't be the same, would it?

I: Now you just look at a thin resistor and not the whole thing, so the length is not L, but just dx. How would you write the resistance for that?

S: Like the integral?

I: What is the resistance dR of just this little resistor?

S: Okay, [writes equation]

$dR = \frac{4\alpha x\, dx}{\pi D^2}$

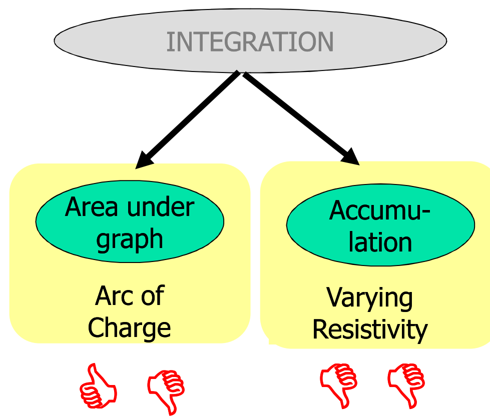
I: Yes, this is the resistance of a thin resistor. Now you want to have the resistance of the whole thing, so … what do you need to do?

S: You need to do the integral then, from 0 to the length of the cylinder. [writes integral]

$R = \int_0^L dR$

Summary: 2nd Semester

- Scaffolding needed to recognize 'area under graph' concept projection of integration.
  - When required to choose appropriate graph, resort to matching integrand with function plotted.
- Significant difficulty using the 'accumulation' concept projection of integration.
  - 'Area under graph' concept projection' may be barrier to learning the 'accumulation' concept projection.



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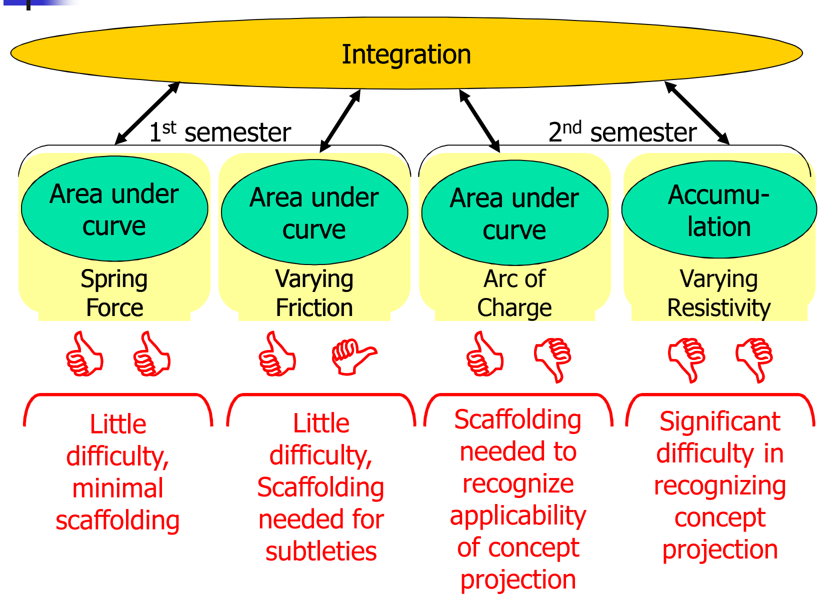
Overall Summary



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Conclusion

Sam (and others in our cohort) have...

- Little difficulty with concept projection of integration as 'area under curve'.
  - Productively used this concept projection in a number of different contexts
  - Needed scaffolding for dealing with subtleties e.g. rescaling and activating concept projection in novel contexts
- Significant difficulty with concept projection of integration as 'accumulation/superposition'
  - Showed evidence that activation of integration as 'area under curve' projection may impede 'accumulation/superposition'

Implications for Instruction

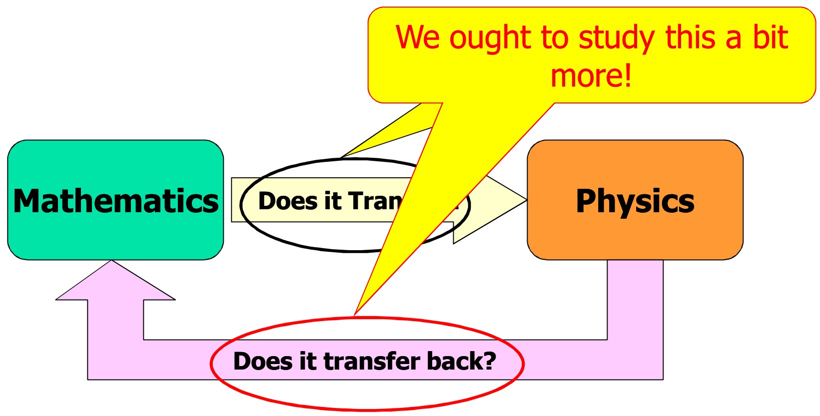
As physics teachers we should …

- facilitate our students' to negotiate different meanings that a math concept can have in different physics contexts.
- gain insights into how our students' learning and problems solving in one physics topic may affect their ability to use math concepts in a different topic.

Implications for Research

As physics education researchers we should…

- collaborate more often with math education researchers.
- investigate how students' experiences in physics courses may affect how they understand math.



Thank You

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