Investigating Trajectories of Learning & Transfer of Problem Solving Expertise from Mathematics to Physics to Engineering

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PROJECT FINDINGS

OVERALL INTERDISIPLINARY FINDINGS

The project activities which resulted in the findings described in this document were guided by our research questions. For convenience of reading, those research questions are repeated here and the findings corresponding to those research questions are reported below that

1. *What kinds of conceptual understanding do students develop and what kinds of difficulties do they encounter when transferring their problem solving skills across problems in different representations in different domains?*

    Our preliminary results are that some students appear to be successful in passing the courses by mastering the skills to carry out each procedure without necessarily developing a deep conceptual understanding. This issue pervades all three disciplines.

    In mathematics, the students have difficulty gaining a conceptual understanding beyond the action and process levels of a mathematical concept. Similarly in physics, students may be able to 'crunch through' the math, but they have difficulty with the physical interpretations of mathematical notations and operations. In particular, although we saw improvement in students understandings of integral as an area underneath a curve, few students displayed an understanding of integral as Chopping & Adding. Similarly, when students move into engineering they appear to have a misdirected understanding of 'area under the curve', properties of functions, time shifts and other concepts that they appear to have had a clear understanding of in their mathematics class.

    The issue however, is not that students have difficulty recalling the concept. Our interviews indicate that they can recall the mathematical concepts quite easily. The issue is deeper, they have difficulty activating the right resources within the contexts of the physics or engineering problem. In other words, they appear to have learned the math concepts insofar as applying them problems in a math course, but later when they transition to a physics course or engineering course, they need considerable scaffolding to do so.

2. *How can we facilitate students' transfer of conceptual understanding and problem solving skills across problems in different representations in different domains?*

    The focus group learning interviews in physics as well as the online exercises in engineering helped us address this issue.

    In the focus group interviews we have created several collections of problems and hints that show some preliminary success in helping students solve problems in multiple representations. Each collection combines abstract mathematical scaffolding, student evaluation of different lines of reasoning, and student creation of complex problems. Using a treatment group – control group, pre-post quasi-experimental design we have show robust improvements in students skills of applying mathematical concepts to physics problems.

Student reactions to the online learning modules in engineering have been generally positive, where the ease of answer entry plays a large role in the experience. Quantitative correlations between module scores, grades on written examinations, and performance in previous mathematics courses have demonstrated variable clarity, but qualitative assessments of the technology-facilitated environment point to a clear increase in student learning and engagement.

Below, we report findings in each of the three disciplinary areas: Mathematics, Physics and Engineering separately, since different activities were carried out in each of these areas. Then we conclude with some overall statements pertaining to our findings in light of the two research questions above.

MATHEMATICS

We have developed (and are continuing to revise) rubrics to rate student conceptual understanding from the interviews using the APOS theory of Dubinsky.

1. Of the 19 students interviewed in October and November, we rated 6 at the Action level, 7 in between the Action and Process level, and 6 at the Process level.

2. Of the 11 students interviewed in February, 2 were still at the Action level, 3 were in between the Action and Process level, 5 were at the Process level, and 1 had advanced past the Process level but had not yet reached the Object level.

3. We are still in the process of analyzing data from the April interviews.

Our preliminary results are therefore that some students are developing deeper conceptual understanding during the courses, but not all. Some students appear to be successful in passing the courses by mastering the skills to carry out each procedure without mastering an understanding of what the mathematics says about a situation.

We have also compared these results to results for conceptual understanding in Differential Equations (i.e. Calculus 4) from interviews done several years earlier as part of a separate project. The current results are in line with results from that study, though we may be seeing more conceptual growth currently than we did then (there have been some instructional changes in the intervening years). We will look at this more closely in the coming year when our current cohort reaches differential equations.

One key issue we are discovering is that students seem to be much more adept at applying graphical information to accumulation problems posed during mathematical interviews than they are during physics interviews. In particular, we have found problems that are mathematically identical where students can do the problem in the context of mathematics but are unable to solve the same problem when posed in the context of physics.

We have two overlapping hypotheses that we are considering to explain this.

i. One is that students' belief system is that the role of mathematics in physics is to provide tools for evaluating equations and therefore they do not access graphical conceptions learned in mathematics in a physics context.
ii. A variation is that students automatically start in a "symbolic" mode when they encounter equations in physics, and their conceptual understanding and problem solving abilities are insufficient to let them shift gears to use graphical tools once they start thinking symbolically.

PHYSICS

Impressions of Individual Interviews

Interview 1: The topic of interview Coulomb's Law. Overall, students had difficulty applying Coulomb's Law for continuous distributions of charge. There was little evidence that students were thinking geometrically about the integrals they were computing. Similarly, some students do not realize that performing the appropriate integral is equivalent to adding up electric fields at point O. There is some evidence that some students have trouble keeping track of the physical meaning of symbols. For example, some students did not know geometrically what the "r" is in Coulomb's Law. Some students had difficulty discussing the symmetry of the physical situation and its computational implications. Additionally, many students had trouble choosing which graph to integrate to find the magnitude of the electric field – many chose a graph based on ease of computation (the one with straight lines) rather than for conceptual reasons. However, students readily recognized that integrals correspond to areas of graphs.

Interview 2: The topic of interview 2 was the resistance of an object. Students continued to have difficulty explaining and applying the geometric meaning of integral although they knew that an integral was needed to calculate the total resistance. Several students set up an incorrect integral by leaving the total length in the integrand and not using a differential length segment. Some students inappropriately commute integration and division, yielding "integral of quotient = quotient of integrals". Students showed improvement in their ability to choose the right graph to integrate. Many students had difficulty using geometric reasoning to find the function of the diameter of the resistor.

Interview 3: The topic for interview 3 was Ampere's Law. Students had serious difficulties solving problems using Ampere's Law. Students had trouble interpreting the meaning of the integral form of the law (e.g. interpreting the dot product and the closed loop integral). The previously identified difficulties interpreting mathematics geometrically proved critical for these problems. Students had trouble selecting an appropriate Amperian loop to perform the computation.

Interview 4: The topic for interview 4 was LRC circuits. The analysis of this interview is currently underway.

Impressions from the Focus Group Interviews

Focus Group Interview 1: The post-test showed that control group, who solved standard textbook problems, scored higher than the treatment group on the equational problem. Only the treatment 2 group scored higher than the control group in the graphical problem.

Focus Group Interview 2: The pre/post assessments revealed a difference between the treatment and control groups at the $p=0.10$ level for calculating the magnitude of a force using Newton's 2nd with a graph. Both the treatment and the control groups had trouble identifying the direction of the force.

Focus Group Interview 3: On the physics aspect, the treatment problems and hints helped improve students' ability to work with energy problems but the improvement was not enough to make a statistically significant difference between the treatment group and the control group. On the representation aspect, our tutorial package also helped improve students' ability to work with graph and equation, and this improvement was large enough to make a statistically significant difference between the treatment group and the control group.

Focus Group Interview 4: On the physics aspect, our the treatment problems and hints helped improve students' ability to work with energy problems but the control problem set also create such an improvement. The improvement in the treatment group was not significantly larger than that in the control group, so there was no significant difference between the treatment group and the control group. On the representation aspect, our tutorial package also helped improve students' ability to work with graph and equation, and this improvement was large enough to make a significant difference between the treatment group and the control group.

Focus Group Interview 5: This comprehensive FOGLI session was designed to test students' ability to work with problems in graphical and equational representations after getting different treatment throughout the semester. In this FOGLI session, students were given three pairs of problems on kinematics, Newton's 2nd law, and work-energy in graphical and equational representations. Results of this FOGLI session indicated that the treatment group scored significantly higher than the control group only in the problems on kinematics. There was no significant difference between two groups in other problems of other topics. This result suggested that although our tutorial packages might have effect on students' performance at the time of the session, they might not have effect on students' retention of what they learned from the FOGLI sessions. This could be explained by the limited time of the treatment.

Overall Impressions

Our overall impressions from all four interviews can be described in terms of the following themes

- *Connecting Abstract Mathematical Notation to the Physical Situation*: We noticed that introductory students have difficulty keeping track of the physical meaning of multiple variables. Students also had difficulty connecting mathematical equations and graphs to the physical situation, particularly with respect to the geometry of the situation (e.g. the locations of charges, distances, etc).
- *Integral as Area Under the Curve*: Students in the longitudinal study displayed an increased understanding that areas under the curves on graph correspond to integrals. Students also showed improvement across interviews in being able to identify the correct graph to use when performing an integral from a graph.
- *Integral as Chopping & Adding*: Students in the second semester of introductory physics also need to thing about integration as chopping up sources and superposing the effects of these sources. Physics students at this level have great difficulty with this understanding of integral.
- *Scaffolding Math Understandings*: Students have difficulty transferring some understandings and skills they achieve from their math courses into their physics courses. We have had some success priming students with an abstract math problem to activate these understandings and then following-up with a related physics problem.
- *Evaluating Other Students' Reasoning*: We have also tried addressing common student difficulties by employing debate problems that ask students to evaluate lines of reasoning from fictitious students. We believe this evaluation process is beneficial to students' conceptual understandings and problem solving skills. While students solve these problems, we have observed conversations among students

that lead to productive understandings and questions; we have seen a reduction in the types of errors that are addressed by these problems.

• *Creating More Complex Problems*: Problems that involve equation and graph representations tend to be more complex than problems that only involve numbers. This added complexity makes these problems more difficult for students. In order to help students become comfortable with this increased complexity, we have students create a complex physics problem from pieces of simple problems and then ask students to outline the solution to their problem.

ENGINEERING

Student reactions to the online learning modules used in ECE 512 – Linear Systems have been generally positive, where the ease of answer entry plays a large role in the experience. Quantitative correlations between module scores, grades on written examinations, and performance in previous mathematics courses have demonstrated variable clarity, but qualitative assessments of the technology-facilitated environment point to a clear increase in student learning and engagement. Instructor benefits are apparent with regard to grading time saved, grading consistency, confidence in student accountability for work submitted, and information regarding when/where students work that is difficult to obtain any other way. Graduate student investment is significant in terms of maintaining the module database and responding to student queries.

Specific findings have come from the 24 interviews performed in the Spring 2010 section of ECE 512 (interview subject: Fourier series):

i. Students do not fully understand the meaning of the mathematical 'integral' and often have a misdirected sense of the 'area under the curve.'

ii. Regarding even/odd functions, students misunderstand their properties (sin(-t) = -sin(t), cos(-t) = cos(t)), they often misapply symmetry when solving for Fourier series coefficients, and they do not realize that an entire integrand must demonstrate even symmetry in order to reduce the integration range by a factor of two.

iii. When working with sinusoids: (a) even/odd symmetry pose problems for students, many of whom also misunderstand the idea that multiplying a sinusoid by -1 yields the same result as a phase shift of pi, and (b) concepts of frequency, angular frequency, and period are still elusive.

iv. Students like to use plots and connect mathematical expressions to plots, but few can make these connections both ways. I.e., function-to-plot transitions are overall good, but plot-to-function transitions cause students difficulties.

v. Students often confuse time and amplitude inversion: f(-t) is assumed to equal -f(t).

vi. Visualization of the DC value of a signal can be difficult.

vii. 'Phase shift' is assumed to mean 'Time shift.'

viii. Some students still struggle with helps such as integral tables and Fourier series references.

ix. For both compact and exponential Fourier series, many students need to revert back to regular trigonometric Fourier series in order to methodically work through a problem. Even in that case, most students can get a correct answer.

x. If one makes changes to a signal (e.g., time inversion), students struggle when predicting the effects that those changes will have on the Fourier series coefficients.

APPENDICES

Several documents have been attached below. These include rubrics developed for analyzing student understanding,, statistical results from focus groups as well as well as talks, posters and papers.