Facilitating Students' Problem Solving Across Multiple Representations in Introductory Mechanics

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1. PHASE 1 – SPRING 2009

1.1 Research Questions

- RQ1: What kinds of difficulty do students encounter while solving physics problems posed in graphical and equational representations?
- RQ2: What kind of scaffolding may help students overcome those difficulties?

1.2 Methodology

- Individual teaching/learning interviews
- 20 student volunteers from a first-semester calculus-based introductory physics course
- Each participant was interviewed four times during the semester.
- Each interview came after an in-class exam.
- In each interview, the students were …
  - Asked to solve three problems
    - Original problem: a problem from the most recent exam
    - Graphical problem: part of the information was given as a graph
    - Equational problem: part of the information was given as a function
  - Asked to think aloud while solving the problems
  - Given verbal hints whenever unable to proceed

1.3 Example of Interview Problems

A hoop radius $r = 1$ cm and mass $m = 2$ kg is rolling at an initial speed v_i of 10 m/s along a track as shown. It hits a curved section (radius $R = 2.0$ m) and is launched vertically at point A. What is the launch speed of the hoop as it leaves the slope at point A?

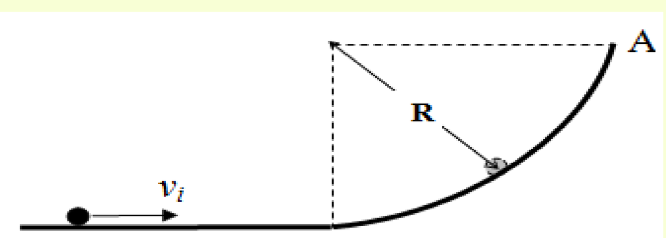
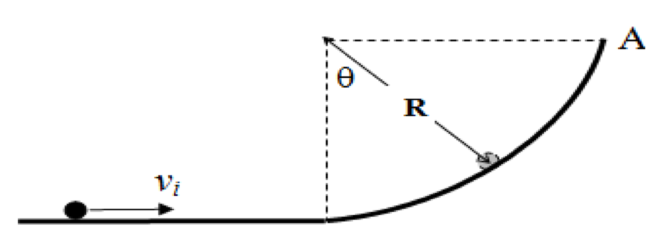


Figure 1. Original problem in interview 4

A sphere radius $r = 1$ cm and mass $m = 2$ kg is rolling at an initial speed v_i of 5 m/s along a track as shown. It hits a curved section (radius $R = 1.0$ m) and is launched vertically at point A. The rolling friction on the straight section is negligible.



The magnitude of the rolling friction force acting on the sphere varies as angle θ as per the graph shown below. What is the launch speed of the sphere as it leaves the curve at point A?

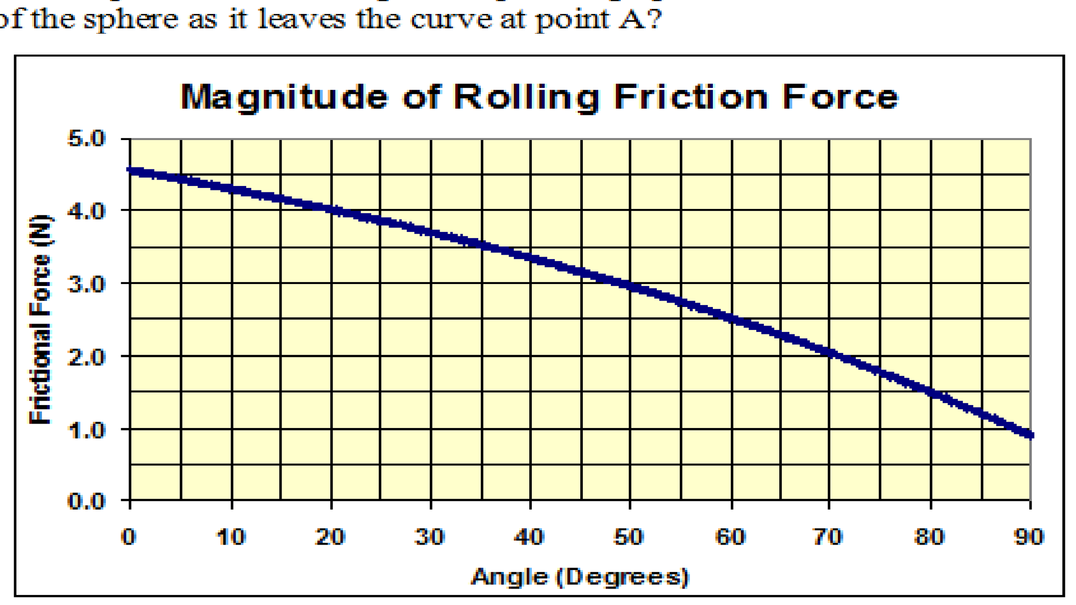
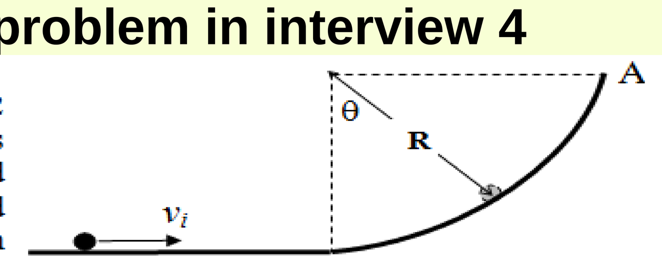


Figure 2. Graphical problem in interview 4

A sphere radius $r = 1$ cm and mass $m = 2$ kg is rolling at an initial speed v_i of 5 m/s along a track as shown. It hits a curved section (radius $R = 1.0$ m) and is launched vertically at point A. The rolling friction on the straight section is negligible.



The magnitude of the rolling friction force F_{roll} (N) acting on the sphere varies as angle θ (radians) as per the following function

$$F_{roll}(\theta) = -0.7\theta^2 - 1.2\theta + 4.5$$

What is the launch speed of the sphere as it leaves the curve at point A?

Figure 3. Equational problem in interview 4

1. PHASE 1 – SPRING 2009 (Cont'd)

1.4 Results

Students' difficulties with the problems in our interviews fall into two categories.

- Difficulties with the physics: inappropriate use of physics principles and concepts.
- Difficulties with the representation:
  - Extracting information from graph
  - Calculating physical quantity from graph/equation
  - Activating required math knowledge in context of physics

Scaffolding (verbal hints) provided by the interviewer.

- Asking students to rethink about physics principles and concepts may help correct students' misunderstanding.
- Guiding students to discuss the physical meaning of mathematical processes may help students activate the correct mathematical knowledge and skills in physics contexts.

2. PHASE 2 – SPRING 2010

2.1 Motivation

Develop sets of research-based exercises targeting the common difficulties observed in phase 1 of the study and test their impact on students' learning to solve physics problems in graphical and equational representations.

2.2 Research Question

Can a research-based sequence of math, physics and non-traditional problems improve students' ability to solve physics problems in graphical and equational representations?

2.3 Methodology

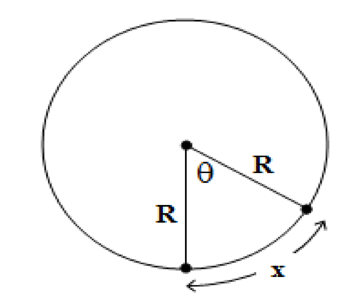
- Focus Group Learning Interview (FOGLI)
- Pre-test/Post-test Control Group Design
- 20 engineering students enrolled in a calculus-based physics course were randomly assigned into either the control group (8 students) or treatment group (12 students)
- Students attempted a pre-test, a problem set prepared by the researchers and a post-test similar to the pre-test.
- Problem set for the treatment group included:
  - two pairs of matched math/physics problems
  - one debate problem
  - two problem posing tasks
- Problem set for the control group included isomorphic textbook problems covering the same topics and principles.
- Students worked individually on the pre-test and post-test, worked in pairs on the problem set.
- Control group provided with printed solution of each problem
- Treatment group required to check-in with a moderator before proceeding to next problem.

2. PHASE 2 – SPRING 2010 (Cont'd)

2.4 Example of a Set of Exercises

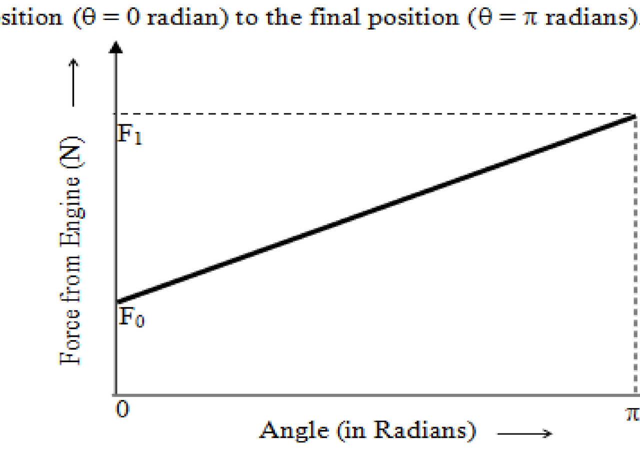
Problem 1

a. What is the length of the arc 'x' along a circle in terms of radius R and angle θ (in radians)?
b. A bug sits on the edge of the turn table of radius R = 2.0 m which is rotating around its center. What is the distance 'x' that the bug has traveled after the turn table has rotated by an angle θ = π/4 ?



Problem 2

A toy plane is attached to a pole by a string and flies around it in a circular arc of radius R (in meters). The graph below shows the force exerted by the engine of the plane as it starts from rest from its initial position (θ = 0 radian) to the final position (θ = π radians).



a. Plot the graph of force of the engine (in Newtons) with respect to the distance 'x' (in meters) that the plane travels along the circular arc from its initial to its final point.
b. Find the work done by engine when the plane travels from its initial point to the final point.

Problem 3

A toy plane is attached to a pole by a string and flies around it in a circular arc of radius R = 3.0 m. The equation below shows the force exerted by the engine of the plane as it starts from rest from its initial position (θ = 0 radian) to the final position (θ = π radians)

$$F(\theta) = a\theta + b$$

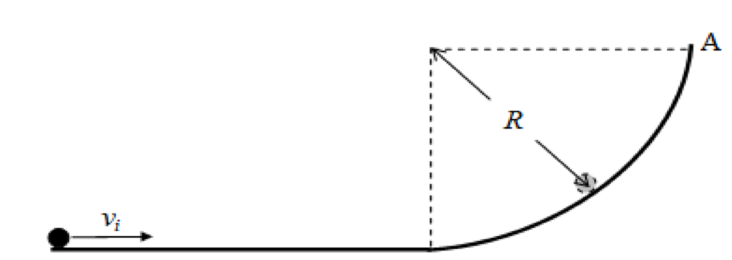
where, a, b are constants; F is in Newtons, and θ is in radians.

a. Write down the equation of force of the engine as a function of the distance 'x' the plane travels along the circular arc from its initial to its final point.
b. Find the work done by the engine when the plane travels from its initial point to the final point in terms of a and b.

Problem 4

Five students are discussing their strategies to solve the following problem.

A hoop radius $r = 1$ cm and mass $m = 2$ kg is rolling at an initial speed v_i of 10 m/s along a track as shown. It hits a curved section (radius $R = 2.0$ m) and is launched vertically at point A.



What is the launch speed of the hoop as it leaves the slope at point A?

Below are parts of the students' strategies. They may not be the complete solutions.
Comment on each student's ideas. Explain who you agree with most and why. For the students who make statements you disagree with, explain what you think is wrong in the student's reasoning.

	Strategy	Comments
David	Energy of the hoop is conserved. On the straight part of the track, the hoop's energy includes both translational and rotational kinetic energy. At point A, the hoop's energy includes potential and translational kinetic energy. When the hoop flies off the track, it does not roll any more, so it does not have rotational kinetic energy at point A.	
Mary	Yes, the hoop does not have rotational energy at point A, but it does not have translational energy on the straight part of the track either. The hoop doesn't have translational motion. It moves forward because it is rolling along the track.	
Eric	The hoop has both translational and rotational motion both on the straight part of the track and at point A. So there are two kinds of kinetic energy in both initial and final energy.	
Susan	Both gravity and normal forces, which are acting on the sphere, do not cause any torque to the sphere so angular momentum of the sphere is conserved between initial point and point A. Angular momentum equals to moment of inertia times angular speed, so I can find angular speed at point A. This angular speed divided by the radius of the sphere is the linear speed of the sphere at point A.	
Jim	I will use kinematics equation: $v^2 = v_0^2 + 2ad$, where a is acceleration due to gravity which is acting on the sphere as it climbs up the track and d is the distance along the track. Then I can find speed of the sphere at point A.	

Problem 5

a. Start with the physics problem in problem 4, modify it by including in it the physics ideas in problem 2 to create a new solvable problem of your own. Write your instructions to solve that new problem.
b. Start with the physics problem in problem 4, modify it by including in it the physics ideas in problem 3 to create a new solvable problem of your own. Write your instructions to solve that new problem.

Figure 4. Problem set for the treatment group in FOGLI session 4

2. PHASE 2 – SPRING 2010 (Cont'd)

- Problems in the pre-test and post-test graded separately on the physics part and the representation part.
- The non-parametric Mann-Whitney test used to test significance of the difference in scores between control and treatment.

TABLE 1. Mann-Whitney for ***physics*** scores

Problem	Pre-test	Post-test
Graph	$p = 0.23$ $z = -1.24$ $r = -0.26$	$p = 0.12$ $z = -1.57$ $r = -0.33$
Equation	$p = 0.19$ $z = -1.31$ $r = -0.28$	$p = 0.07$ $z = -1.80$ $r = -0.38$

- Table 1 : Treatment does not appear to improve students' ability to solve work-energy problems compared to the control.

TABLE 2. Mann-Whitney for ***representation*** scores

Problem	Pre-test	Post-test
Graph	$p = 0.20$ $z = -1.29$ $r = -0.28$	$p = 0.04$ $z = -2.07$ $r = -0.44$
Equation	$p = 1.00$ $z = -0.00$ $r = -0.00$	$p = 0.01$ $z = -2.65$ $r = -0.56$

- Table 2 : Score on representation aspect of the treatment group is not statistically significantly higher than that of the control group on the pre-test, but it is statistically significantly higher in the post-test.
- Treatment problem set improves students' ability to work with graphical and equational representations more than the control problem set does.

3. CONCLUSIONS

- Students' difficulties with….
  - Physics of the problems were due primarily to students' misunderstanding or misuse of physical principles and concepts,
  - Representation - graphical and equational - were due to students' inability to activate the appropriate mathematical knowledge in physics contexts.
- Research-based sequence of problems…
  - has a positive effect in improving students' performance on the representation aspect of problems, but
  - it is not as effective in improving students' performance on the physics aspect of problems.