Facilitating Strategies for Solving Work-Energy Problems in Graphical and Equational Representations

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1. MOTIVATION

- In our previous study [1], we found that:
  - Students encountered a variety of difficulties when solving problems in graphical and equational representations.
  - These difficulties were primarily due to students' inability to activate the required mathematical knowledge in the context of a physics problem.
- In this study:

  We developed problem sets aimed at facilitating the activation of required mathematical skills to solve physics problems in graphical and equational representations.

2. RESEARCH QUESTION

Can a research-based sequence of math, physics and non-traditional problems improve students' ability to solve physics problems in graphical and equational representations?

3. METHODOLOGY

Focus Group Learning Interviews (FOGLI's) [2]

- Pre-test/post-test Control Group Design
- 20 engineering students enrolled in a calculus-based physics course were randomly assigned into either a control group (8 students) or treatment group (12 students)
- Students attempted a pre-test, a problem set prepared by the researchers and a post-test similar to the pre-test.
- Problem set for the treatment group included:
  - two pairs of matched math and physics problems
  - one debate problem
  - two problem posing tasks [3]
- Problem set for the control group included isomorphic textbook problems covering the same topics and principles.
- Students worked individually on the pre-test and post-test and worked in pairs on the problem set.
- Students in the control group were provided with a printed solution of each problem while students in the treatment group were required to check in with a moderator before proceeding to the next problem.

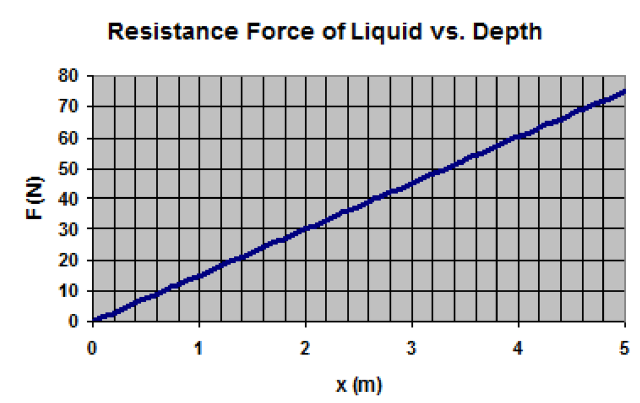
4. INTERVIEW PROBLEMS

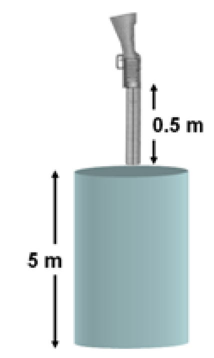
Problem A

A 0.05 kg bullet is loaded into a gun compressing a spring which has spring constant k = 5000 N/m. The gun is tilted vertically downward and the bullet is fired into a drum 5.0 m deep, filled with a liquid.

The barrel of the gun is frictionless. The magnitude of the resistance force provided by the liquid changes with depth as shown in the graph below. The bullet comes to rest at the bottom of the drum.

What is the spring compression x?





Problem B

A 0.05 kg bullet is loaded into a gun compressing a spring which has spring constant k = 5000 N/m. The gun is tilted vertically downward and the bullet is fired into a drum 5.0 m deep, filled with a liquid.

The barrel of the gun is frictionless. The magnitude of the resistance force F (in Newtons) provided by the liquid changes with depth x (in meters) as per the following function:

$$F(x) = 8x + 0.5x^2$$

The bullet comes to rest at the bottom of the drum.
What is the spring compression x?

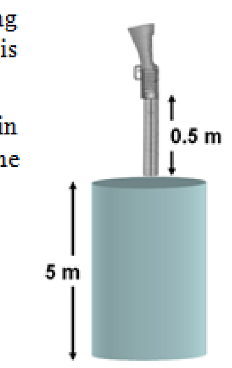
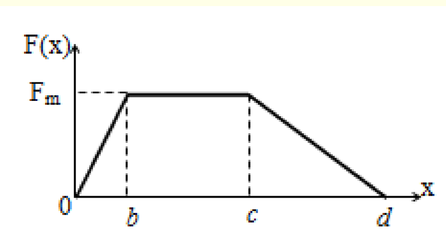


Figure 1. Problems in the pre- and post-test

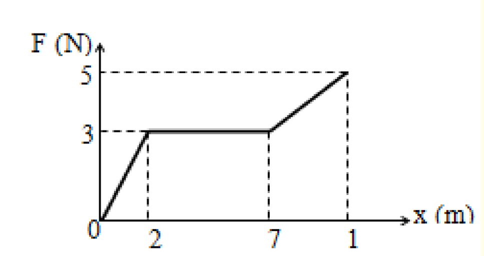
Problem 1

The graph below shows the magnitude of a force F(x) acting on an object with respect to the displacement x of the object (F is in Newtons and x is in meters). Find the work done by force F on the object over the distance d that the force is acting.



Problem 2

The graph below shows the magnitude of a force F (in Newtons) acting on an object with respect to the displacement x (in meters) of the object. Find the work done by force F on the object over the displacement from 0 m to 10 m.



Problem 3

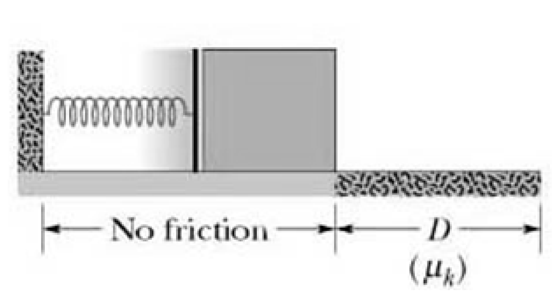
A block is pulled on a horizontal frictionless floor by a force whose magnitude F (in Newtons) depends on the displacement x of the block (in meters) as per the function: $F(x) = ax^2 + bx + c$ (a, b, c are constants). Find the work done by force F when the block has been moved from x_1 to x_2.

Problem 4

A block is pulled on a horizontal frictionless floor by a force F whose magnitude depends on the displacement of the block as per the function: $F(x) = 2x^3 - 3x + 2$ (x is in meters, F is in Newtons). Find the work done by force F when the block has been moved from 0 m to 2 m.

Problem 5

A 3.5 kg block is accelerated from rest by a spring, spring constant 632 N/m that was compressed by an amount x. After the block leaves the spring it travels over a horizontal floor with a coefficient of kinetic friction $\mu_k = 0.25$. The frictional force stops the block in distance D = 7.8 m.



What was the spring compression x?

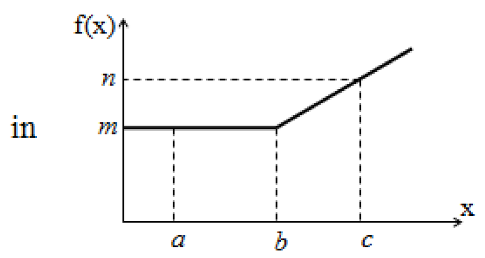
Figure 2. Problem set for the control group in FOGLI session 3

4. INTERVIEW PROBLEMS (Cont'd)

Problem 1

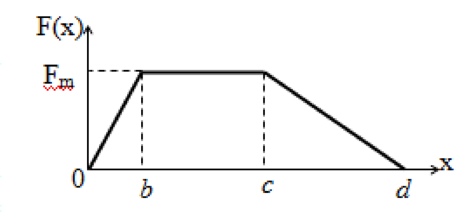
The graph of a function f(x) is given below.

Find the value of the integral $\int_a^c f(x)\,dx$ in terms of the constants a, b, c, m, n.



Problem 2

The graph below shows the magnitude of a force F(x) acting on an object with respect to the displacement x of the object (F is in Newton and x is in meters). Find the work done by force F on the object over the distance d that the force is acting.



Problem 3

Find the area of the region surrounded by the graphs of the following functions: $f(x) = x^3 + 2x + 1$, $f(x) = 0$, $x = x_1$, $x = x_2$.

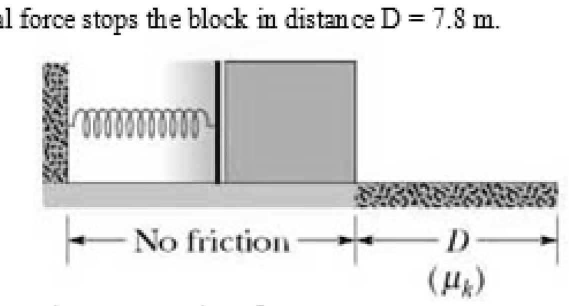
Problem 4

A block is pulled on a horizontal frictionless floor by a force F whose magnitude (in Newton) depends on the displacement x of the block (in meters) as per the function: $F(x) = ax^2 + bx + c$ (a, b, c are constants). Find the work done by force F when the block has been moved from x_1 to x_2.

Problem 5

Five students are discussing their strategies to solve the following problems.

A 3.5 kg block is accelerated from rest by a spring, spring constant 632 N/m that was compressed by an amount x. After the block leaves the spring it travels over a horizontal floor with a coefficient of kinetic friction $\mu_k = 0.25$. The frictional force stops the block in distance D = 7.8 m.



What was the spring compression x?

Which student is correct? Comment on each student's ideas. Explain who you agree with most and why. For the students who make statements you disagree with, try to identify what went wrong in the student's reasoning.

Student	Strategy	Comments
David	Energy is conserved so all the changes in energy add to zero. The block starts from rest and then comes to a stop, so there is no change in kinetic energy. The only energy that changes is the spring's potential energy and that's good because that involves the compression of the spring. You can calculate the change in potential energy and solve for the compression.	
Mary	Friction is involved so you need to use ΔK + ΔU = W, where W = -μkmgD is the work done by friction. ΔK is zero because initial and final speeds are zero. The initial U is that of the spring and final U is zero. Then put everything into the equation and solve for x.	
Eric	Isn't the work +μkmgD, because W in that equation is the amount of work done and therefore it must be positive?	
Susan	But the spring does work on the block too and you have to take that into account. Work is force times distance, and since the force of the spring is -kx and the spring pushes the block a distance x, the work done by the spring is -kx^2. That's the formula you should use to find the compression.	
Mike	All you have to do to calculate the work done by the spring is to plug in the total distance the spring pushes the block into the force -kx. So, if the initial compression is L, the work done by the spring is -kL.	

Problem 6

a. Start with the physics problem in problem 5, modify it by including in it the physics ideas in problem 2 to create a new solvable problem of your own. Write your instructions to solve that new problem.
b. Start with the physics problem in problem 5, modify it by including in it the physics ideas in problem 4 to create a new solvable problem of your own. Write your instructions to solve that new problem.

Figure 3. Problem set for the treatment group in FOGLI session 3

5. RESULTS

- Problems in the pre-test and post-test graded separately on the physics part and the representation part.
- The non-parametric Mann-Whitney test used to test significance of the difference in scores between control and treatment groups.

TABLE 1. Mann-Whitney for ***physics*** scores

Problem	Pre-test	Post-test
Graph	p = 1.00 z = - 0.04 r = - 0.01	p = 0.42 z = - 0.85 r = - 0.19
Equation	p = 0.82 z = –0.27 r = –0.06	p = 0.51 z = –0.69 r = –0.16

- Table 1 : Treatment does not appear to improve students' ability to solve work-energy problems compared to the control.

TABLE 2. Mann-Whitney for ***representation*** scores

Problem	Pre-test	Post-test
Graph	p = 0.79 z = 0.31 r = - 0.07	*p = 0.04* z = - 2.04 r = - 0.46
Equation	p = 0.88 z = 0.11 r = 0.03	*p = 0.05* z = - 1.97 r = - 0.44

- Table 2 : Score on representation aspect of the treatment group is not statistically significantly higher than that of the control group on the pre-test, but it is statistically significantly higher on the post-test.
- Treatment problem set improves students' ability to work with graphical and equational representations more than the control problem set does.

6. CONCLUSIONS

Initial results suggest that our research-based sequence of problems has a positive effect in improving students' performance on the representation aspect of problems, while it is not as effective in improving students' performance on the physics aspect of problems.

7. REFERENCES

1. D. Nguyen and N. S. Rebello, in 2009 Physics Education Research Conference, edited by M. Sabella, C. Henderson, and C. Singh (AIP, Ann Arbor, MI, 2009), Vol. 1179.
2. F. A. Mateycik, Ph.D. Dissertation, Kansas State University, 2010.
3. J. P. Mestre, Journal of Applied Developmental Psychology 23 (1), 9 (2002).
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