Students' Difficulties with Integration in Introductory Electricity and Magnetism

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INTRODUCTION

Motivation

Investigate the difficulties students have when solving E&M problems involving integration.

Methodology

- Individual teaching/learning interview.
- 15 student volunteers from a calculus-based introductory E&M course.
- Each student was interviewed four times during the course.
- Each interview came after an exam in the course.
- In each interview, the students were:
  - Asked to solve three to five problems posed in numerical, graphical and equational representations on the topics covered in the most recent exam.
  - Asked to think aloud while solving the problems.
  - Given verbal hints whenever they made a mistake or were unable to proceed.
- In this study, we only consider the equational problems.

Physics problem solving with integration

Solving a physics problem involving integration can be divided into four steps:

- recognize the need for an integral
- set up the expression for the infinitesimal quantity
- accumulate the infinitesimal quantity
- compute the integral

We will discuss students' difficulty in each of these four steps.

THE INTERVIEW PROBLEMS

Interview 1

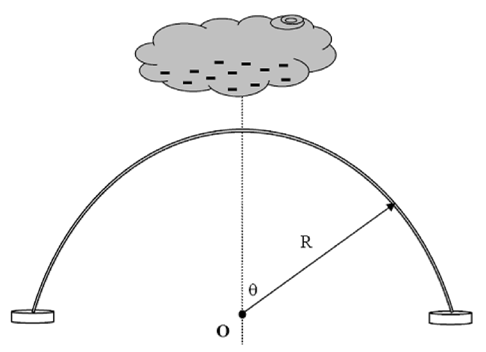
The arch problem

You are standing at the center of a non-conducting circular arch of radius R in a stormy day. There are negatively charged clouds over the arch. The charge distribution λ on the arch now depends on the angle θ as per the function:

$$\lambda(\theta) = \lambda_0 \cos\theta$$

where λ_0 is a positive constant.

Find the magnitude and direction of the electric field at your feet (i.e. at a point O on the ground directly below the top of the arch).



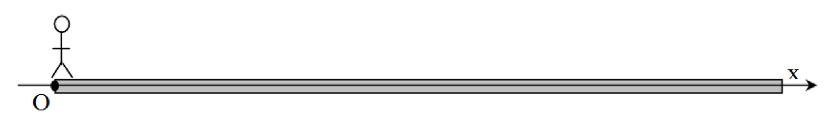
The rod problem

A straight metal rod of length L is lying on the ground but is insulated from the ground. The charge on the rod is distributed with charge density given as per the following function:

$$\lambda(x) = \alpha x^2$$

where: α is a positive constant, x is the position on the x-axis relative to the origin O as shown in the figure below.

Find the magnitude and direction of the electric field at your feet, located at x = 0.



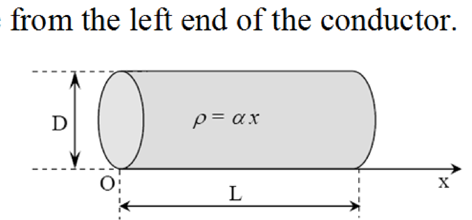
Interview 2

The cylindrical conductor problem

Find the resistance of a cylindrical conductor of length L, diameter D. The resistivity $\rho(x)$ is changing along the conductor as per the following function:

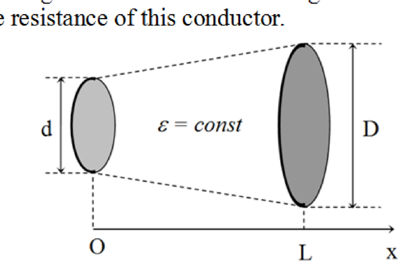
$$\rho(x) = \alpha x$$

where x is the distance from the left end of the conductor.



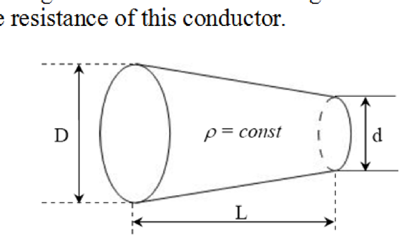
The capacitor problem

A conductor has diameter decreasing from D to d over its length L. The resistivity ρ is constant along the length of this conductor. Find the resistance of this conductor.



The truncated-cone conductor problem

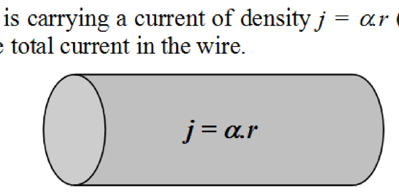
A conductor has diameter decreasing from D to d over its length L. The resistivity ρ is constant along the length of this conductor. Find the resistance of this conductor.



Interview 3

The current problem

A cylindrical wire of radius R is carrying a current of density $j = \alpha . r$ (α is a constant, r is the distance from the center of the wire). Find the total current in the wire.



FINDINGS

	Recognizing the need for an integral	Setting up expression for infinitesimal quantity	Accumulating the infinitesimal quantities	Computing the integral
The arch problem	All students integrated	All 15 students had correct expression for dE.	• 7 students integrated the y-component of dE. • The other 8 students integrated the whole dE.	13 students could not recall the relation dq = λds.
The rod problem	All students integrated	All 15 students had correct expression for dE.	All students integrated dE.	11 students did not recognize that "ds = dx" or interpreted "r" in Coulomb's law as radius.
The cylindrical conductor problem	3 students did not integrate	• 4 students had correct expression for dR. • The other 11 students wrote $dR = \rho(x)\frac{L}{A}$ or $dR = \rho(x)\frac{Ldx}{A}$	All students integrated dR.	All students were able to compute the integral.
The truncated-cone conductor problem	All students integrated	• 13 students had correct expression for dR. • The other 2 students wrote $dR = \frac{\rho L}{dA}$	All students integrated dR.	• One student set the limits from d to D. • None of the students were able to compute the integral themselves.
The capacitor problem	All students integrated	• 10 out of 12 students had correct expression for dC. • The other 2 students wrote $dC = \varepsilon\frac{dA}{L}$	• 2 out of 12 students integrated 1/dC. • The other 10 students integrated dC.	None of the students were able to compute the integral themselves.
The current problem	2 students did not integrate	• 2 students had correct expression for dI. • The other 13 students wrote $dI = Aj(r)$ or $dI = Aj(r)dr$	All students integrated dI.	All students were able to compute the integral.

CONCLUSION

- Most of the students did not have significant difficulty recognizing the need for integration in solving the problems. We observed that the non-constant physical quantity given in the problem statement was the major cue for integration.
- Many students in our interviews ignored the infinitesimal term "dx" (or "dr", "dθ") or simply appended it to the formula for the constant case or to a quantity that was changing - actions that essentially change the physical meaning of the expression for the infinitesimal quantity.
- After having the correct expression for the infinitesimal quantity, almost all students started integrating that expression without noticing how these quantities should be added up.
- Students also had difficulties converting one variable to another variable, determining the limits of the integral, interpreting the physical meaning of symbols, and performing algebraic computation of the integrals.