TUTORIALS TO FACILITATE STUDENTS' REPRESENTATIONAL SKILLS FOR PROBLEM SOLVING IN INTRODUCTORY COLLEGE PHYSICS

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Abstract

In a previous study, we found that students in introductory physics encountered a variety of difficulties when solving physics problems posed in graphical and equational representations. However, students were eventually able to solve those problems with appropriate scaffolding provided by the facilitator. Based on the knowledge of the difficulties students encountered and the scaffolding that might be helpful, we developed, tested and refined tutorial materials to facilitate students' problem solving with physics problems in graphical and equational representations on several topics of introductory physics. In this paper, we present the tutorial material on the topic of work-energy and its impact on students' performance on problems in graphical and equational representations.

OBJECTIVES

Our previous study (Authors, 2009) indicated that students encountered a variety of difficulties when solving work-energy problems posed in graphical and equational representations. The difficulties could be attributed primarily to students' inability to activate the appropriate knowledge of mathematical representations, particularly graphs and equations, in a physics context. These findings suggested that exercises that facilitate the activation of the appropriate mathematical representation skills may help students improve their performance on problems.

In this study, we developed and tested tutorials to facilitate students' solving physics problems in graphical and equational representations in introductory physics. Each tutorial contained a sequence of paired math and physics problems with a verbal protocol used by the instructor to facilitate students' learning.

Our research question is: To what extent does our tutorial help students improve their performance on physics problems posed in graphical and equational representations?

THEORETICAL FRAMEWORK

Our study is based on a theoretical framework that focuses on two aspects of transfer: horizontal and vertical (Authors, 2007), which are akin to low-road and high-road transfer (Salomon & Perkins, 1989).

Horizontal transfer involves the application of well developed knowledge to new situations. A learner possesses a well developed schema for solving a problem which invoked when the problem is encountered. The learner 'plugs-in' information from the problem into the schema. An example of horizontal transfer occurs when solving a simple 'plug-n-chug' problem.

Vertical transfer occurs when a learner encounters a problem that cannot be solved using an existing schema. Then they must adapt and reconstruct their schema, incorporating new knowledge to solve the problem. Scaffolding is often needed to facilitate vertical transfer.

The ability of a learner to creatively adapt to a new problem is adaptive expertise (Schwartz, Bransford, & Sears, 2005). To gain adaptive expertise a learner must navigate a sequence learning experiences involving vertical and horizontal transfer (See Figure 1)

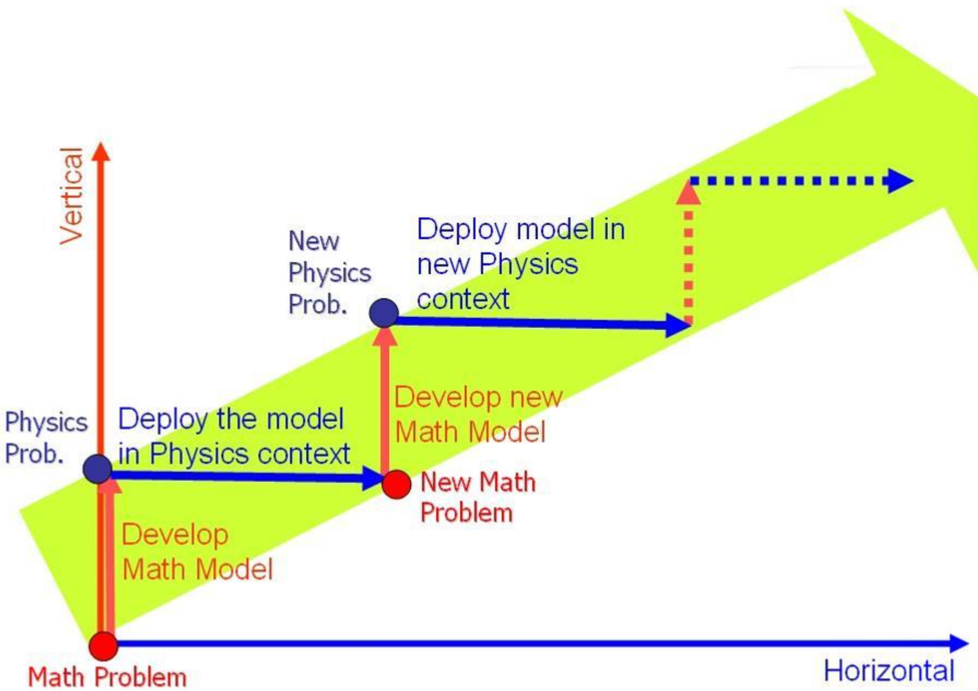


Figure 1. Theoretical Framework showing sequences of vertical and horizontal transfer needed to achieve adaptive expertise.

The method below is an implementation of our framework. The tutorial includes sequences of math and physics problems. The math problems provide opportunities to develop representational models i.e. they involve vertical transfer. The physics problems provide the opportunity to use these models i.e. they involve horizontal transfer.

METHOD

The pretest-posttest control group experimental design (Campbell & Stanley, 1963) was used. Twenty five volunteers were randomly assigned into either a control or a treatment group. The number of students varied with each session, ranging from eight to 10 students in the control group and 12 to 14 students in the treatment group. Most of the students were freshmen or sophomores in engineering majors.

Students in both groups participated in focus group learning interviews (FOGLI's) where they worked in groups of 2-3 students each, and interacted with the facilitator (Mateycik, 2010). we conducted five FOGLI sessions, each after an exam in the course. In each of these 90-minute sessions, for the first 15-20 minutes students individually attempted a pre-test consisting of a graphical and an equational problem.

In the next 40–50 minutes, students in the treatment group worked on our tutorials which included pairs of matched math and physics problems while the control group worked on isomorphic textbook problems covering the same concepts and principles. Students in both groups discussed their problem solving strategies with their partners. Students in the control group were provided with a printed solution of each problem before proceeding to the next problem. Students in the treatment group were required to check-in with a facilitator before proceeding to the next problem. The facilitator elicited students' ideas and facilitated them to solve the tutorial problems. During the conversation, the facilitator helped students recognize and correct their errors in their solutions. In the last 15-20 minutes, students individually attempted the post-test which differed from the pre-test in numerical values in the problem statements.

DATA SOURCES

Students' pre-tests, post-tests and the tutorial worksheets were collected. Rubrics were created to grade the pre-test and post-test problems (transfer tasks) in each FOGLI session. Each problem was graded separately on the physics aspect and the representation aspect. The maximum score on the physics aspect was 10 points and on the representation aspect was 8 points. The rubric for the physics aspect rated five dimensions: *approach* (i.e. was correct principle used?), *equations* (i.e. were correct equations used?), *values* (i.e. were correct values of quantities used?), *manipulation* (i.e. were the equations correctly manipulated?), and *units* (i.e. were the units correct?). The rubric for the representation aspect also rated five dimensions: *gathering* (i.e. was the correct information gathered from the representation?), *mapping* (i.e. was the information correctly mapped on to the physics problem), *setting up* (i.e. was the information correctly used in the physics problem?), *manipulation* and *units* which are same as above.

Due to the small number of participants in each group, the non-parametric Mann-Whitney U Test (Field, 2009) was employed to test the significance of the difference between the scores of two groups on the pre-test and post-test. The null hypothesis was that the scores of the two groups were not statistically significantly different.

RESULTS AND DISCUSSION

In this paper, we present the tutorials on the topic of Work-Energy in introductory mechanics and their impact on students' performance on Work-Energy problems posed in graphical and equational representations. There were two tutorial packages on Work-Energy: one on Work-Energy in linear motion (used FOGLI session 3) and the other on Work-Energy in rotational motion (used in FOGLI session 4). The problems in these two tutorials followed the same sequence.

In this section, we present the scores of each group in each problem and the results of the Mann-Whitney tests in each of the FOGLI sessions 3 and 4.

FOGLI session 3 – Work-Energy in Linear Motion

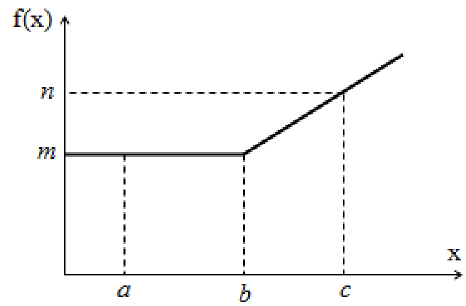
There were eight students in the control group and 12 students in the treatment group in this FOGLI session. The inter-rater reliability was 92% for the rubric on the physics aspect and was 89% for the rubrics on the representation aspect of pre- and post-test problems in this FOGLI session.

A pair of matched math and physics problems in our tutorial used in FOGLI session 3 is presented in Figure 2. The protocols for those problems are presented in Figure 3.

Problem 1

The graph of a function f(x) is given below.

Find the value of the integral $\int_a^c f(x)\,dx$ in terms of the constants a, b, c, m, n.



Problem 2

The graph below shows the magnitude of a force F(x) acting on an object with respect to the displacement x of the object (F is in Newtons and x is in meters). Find the work done by force F on the object over the distance d that the force is acting.

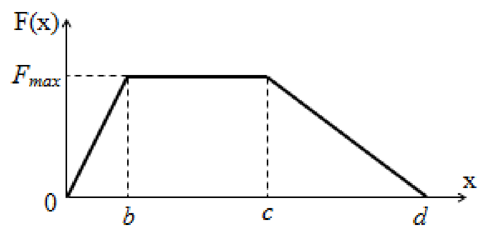


Figure 2. A pair of matched math and physics problems in the tutorial used in FOGLI session 3

Protocol for problem 1

- Ask students to explain how they have solved the problem.
- If students do the problem correctly, then ask them:
  - What did you learn from this problem?
  - Give an example of a physics situation in which you can apply what you've learned from this problem.
- If students don't know how to do the problem or do it incorrectly, then ask them:
  - Now you have to evealuate an integral and you are given a graph. What mathematical knowledge can you use?
  - What is the relation between an integral of a function and the graph of that function?

Protocol for problem 2

- Ask students to explain how they have solved the problem.
- If students do the problem correctly, then ask them:
  - How did you apply what you learned in problem 1 to this problem?
  - What did you learn from this problem?
- If students don't know how to do the problem or do it incorrectly, then ask them:
  - Is the force constant?
  - What is the equation to calculate work done by a non-constant force?
  - How can you use this graph to calculate work according to that equation? Think of what you learned from problem 1.
  - What did you learn from this problem?

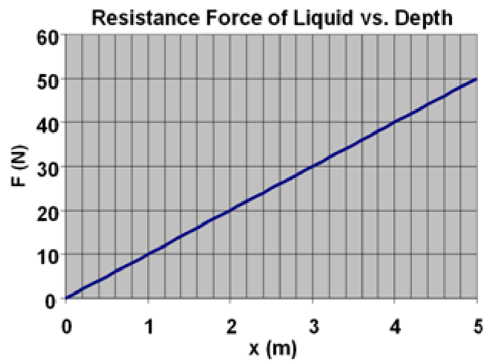
Figure 3. Protocol for the problems in Figure 3.

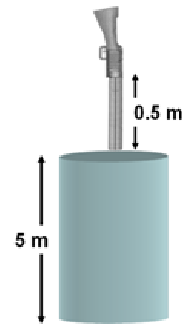
The post-test question (transfer task) used in FOGLI 3 is shown in Figure 4. The pre-test question was similar to the post-test question, except with changed numerical values.

Graph problem

A 0.1 kg bullet is loaded into a gun compressing a spring which has spring constant k = 6000 N/m. The gun is tilted vertically downward and the bullet is fired into a drum 5.0 m deep, filled with a liquid.

The barrel of the gun is frictionless. The magnitude of the resistance force provided by the liquid changes with depth as shown in the graph below. The bullet comes to rest at the bottom of the drum.



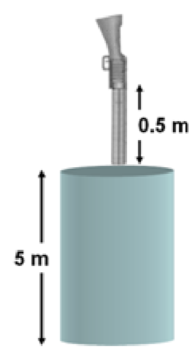


What is the spring compression x?

Equation problem

A 0.1 kg bullet is loaded into a gun compressing a spring which has spring constant k = 6000 N/m. The gun is tilted vertically downward and the bullet is fired into a drum 5.0 m deep, filled with a liquid.

The barrel of the gun is frictionless. The magnitude of the resistance force F (in Newtons) provided by the liquid changes with depth x (in meters) as per the following function:



$$F = 10x + 0.6x^2$$

The bullet comes to rest at the bottom of the drum

What is the spring compression x?

Figure 4. Post-test problems in FOGLI session 3

Physics aspect: The means and standard deviations of the physics scores of each group in the pre-test and post-test are presented in Table 1. The Mann-Whitney test result is presented in Table 2.

Table 1. ***Physics*** score out of 10: Mean (± S.D.)

Problem	**Group**	**Pre-test**	**Post-test**
Graph	Control	8.25 (± 2.25)	7.88 (± 2.80)
	Treatment	8.08 (± 2.78)	9.08 (± 1.31)
Equation	Control	8.13 (± 2.59)	8.50 (± 2.00)
	Treatment	8.33 (± 2.27)	9.17 (± 1.11)

Table 2. Mann-Whitney test result for ***physics*** scores.

Problem	**Pre-test**	**Post-test**
Graph	U = 48.5, p = 1.00, z = –0.04, r = –0.01	U = 59.0, p = 0.42, z = –0.85, r = –0.19
Equation	U = 51.5, p = 0.82, z = –0.27, r = –0.06	U = 57.0, p = 0.51, z = –0.69, r = –0.16

Table 2 indicates that there was no statistically significant difference in physics scores between the control group and the treatment group in any problem of the pre-test and post-test. Although the effect sizes were slightly higher in the post-test (r = - 0.19 in the Graph problem and r = - 0.16 in the Equation problem) than in the pre-test (r = - 0.01 and r = - 0.06 respectively), the effects were still weak. This implies that our tutorial in this session didn't improve students' ability to solve work-energy problems compared to the control problem set. This result might suggest that the treatment should be refined to increase students' practice with the underlying physics processes of the problems.

Representation aspect: The means and standard deviations of the representation scores of each group in the pre-test and post-test are presented in Table 3. The Mann-Whitney test result is presented in Table 4.

Table 3. ***Representation*** score out of 8: Mean (± S.D.)

Problem	**Group**	**Pre-test**	**Post-test**
Graph	Control	4.88 (± 2.75)	6.13 (± 1.89)
	Treatment	5.33 (± 2.84)	7.58 (± 0.90)
Equation	Control	4.25 (± 2.82)	4.88 (± 2.80)
	Treatment	4.08 (± 2.64)	7.00 (± 1.60)

Table 4. Mann-Whitney test result for ***representation*** scores.

Problem	**Pre-test**	**Post-test**
Graph	U = 52.0, p = 0.79, z = 0.31, r = -0.07	U = 74.5, *p = 0.04*, z = -2.04, r = -0.46
Equation	U = 46.5, p = 0.88, z = 0.11, r = 0.03	U = 73.5, *p = 0.05*, z = -1.97, r = -0.44

These tables indicate that the representation score of the treatment group was not statistically significantly different from that of the control group in the pre-test, but it was statistically significantly higher in the post-test ($p < .05$). The effect sizes, r = -0.46 in the graph problem and r = -0.44 in the equation problem in the post-test suggest that these were strong effects. This result implies that our tutorial in session 3 significantly improved students' ability to work with graphical and equational representations more than the control problem set did.

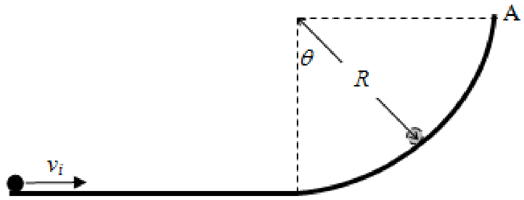
FOGLI session 4 – Work-Energy in Rotational Motion

There were nine students in the control group and 13 students in the treatment group in this FOGLI session. The inter-rater reliability was 88% for the rubric on the physics aspect and was 95% for the rubrics on the representation aspect of pre- and post-test problems in this FOGLI session.

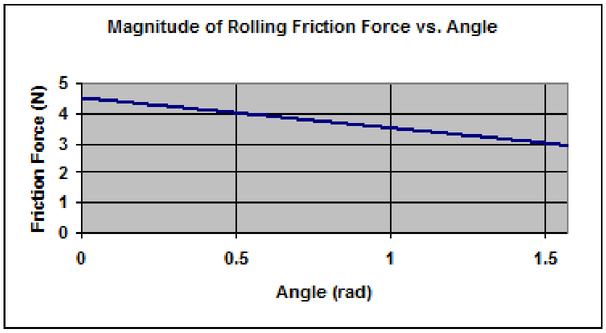
The post-test question (transfer task) used in FOGLI 4 is shown in Figure 5. The pre-test question was similar to the post-test question, except with changed numerical values.

Graph problem

A sphere radius r = 1 cm and mass m = 2 kg is rolling at an initial speed v_i = 5 m/s along a track as shown. It hits a curved section (radius R = 1.0 m) and is launched vertically at point A. The rolling friction on the straight section is negligible.



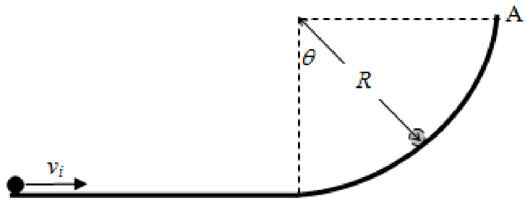
The magnitude of the rolling friction force acting on the sphere varies with angle θ as per the graph shown below



What is the launch speed of the sphere as it leaves the curve at point A?

Equation problem

A sphere radius r = 1 cm and mass m = 2 kg is rolling at an initial speed v_i = 5 m/s along a track as shown. It hits a curved section (radius R = 1.0 m) and is launched vertically at point A. The rolling friction on the straight section is negligible.



The magnitude of the rolling friction force F_{roll} (N) acting on the sphere varies with angle θ (radians) as per the following function

$$F(\theta) = 4.5 - 1.2\theta$$

What is the launch speed of the sphere as it leaves the curve at point A?

Figure 5. Post-test problems in FOGLI session 4

Physics aspect: The means and standard deviations of the physics scores of each group in the pre-test and post-test are presented in Table 5. The Mann-Whitney test result is presented in Table 6.

Table 5. ***Physics*** score out of 10: Mean (± S.D.)

Problem	Group	Pre-test	Post-test
Graph	Control	4.89 (± 3.66)	7.00 (± 3.04)
	Treatment	6.54 (± 3.57)	8.77 (± 1.09)
Equation	Control	3.78 (± 3.31)	5.11 (± 4.31)
	Treatment	6.08 (± 3.95)	8.62 (± 1.39)

Table 6. Mann-Whitney test result for ***physics*** scores.

Problem	Pre-test	Post-test
Graph	U = 48.5, p = 1.00, z = –0.04, r = –0.01	U = 59.0, p = 0.42, z = –0.85, r = –0.19
Equation	U = 51.5, p = 0.82, z = –0.27, r = –0.06	U = 57.0, p = 0.51, z = –0.69, r = –0.16

These tables show a similar trend of the physics score in FOGLI session 4 as in FOGLI session 3, so the conclusions are the same: our tutorial in this session didn't improve students' ability to solve work-energy problems compared to the control problem set. The treatment should be refined to increase students' practice with the underlying physics processes of the problems.

Representation aspect: The means and standard deviations of the representation scores of each group in the pre-test and post-test are presented in Table 7. The Mann-Whitney test result is presented in Table 8.

Table 7. ***Representation*** score out of 8: Mean (± S.D.)

Problem	Group	Pre-test	Post-test
Graph	Control	2.00 (± 2.45)	3.78 (± 2.68)
	Treatment	3.08 (± 2.56)	5.92 (± 2.81)
Equation	Control	3.22 (± 2.22)	4.56 (± 2.24)
	Treatment	3.54 (± 1.45)	7.00 (± 1.53)

Table 8. Mann-Whitney test result for ***representation*** scores.

Problem	Pre-test	Post-test
Graph	U = 40.0, p = 0.20, z = -1.29, r = -0.28	U = 28.0, *p = 0.04*, z = -2.07, r = -0.44
Equation	U = 58.5, p = 1.00, z = -0.00, r = -0.00	U = 20.0, *p = 0.01*, z = -2.65, r = -0.56

These tables also indicate similar trend in the representation score as in FOGLI session 3, so the same conclusions apply: our tutorial in session 4 significantly improved students' ability to work with graphical and equational representations more than the control problem set did.

Similar results obtained from two FOGLI sessions with two tutorials on Work-Energy containing the same sequence of problems and similar protocols appear to indicate that the tutorial package containing pairs of matched math and physics problems, debate problem and problem posing tasks seems to help students improve their ability to work with graphical and equational representations while it still needs to be refined to help students more with the physics of the problems.

The main limitations of the study reported here is the small sample size of students with whom these tutorial materials were implemented. In future implementations, we plan to scale up the study to include a larger sample size.

SIGNIFICANCE OF WORK

This paper speaks to the conference theme – 'Inciting the Social Imagination: Education Research for the Public Good' An important aspect of education for the 'public good' is to enable learners to develop the cognitive tools for the 21st Century. The ability to solve problems encountered in different representations -- representational fluency, is an important skill for the citizenry of tomorrow. The significance of this study is that it provides proof-of-concept that appropriately designed sequences of problems that facilitate transfer can improve students' problem solving skills with regard to their representational fluency in solving these problems.

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