Characterizing Student Use Of Differential Resources In Physics Integration Problems

Dehui Hu and N. Sanjay Rebello

Department of Physics, 116 Cardwell Hall, Kansas State University, Manhattan, KS 66506-2601

Abstract. Developing the skills to set up integrals is critical for students' success in calculus-based physics courses. It requires a high level understanding of both math and physics concepts. Previous studies have shown that students encounter a lot of difficulties when setting up integrals in the context of electricity and magnetism. However, the causes of students' difficulties have not been carefully studied in the past. In order to understand students' solutions and mistakes from a resources perspective, we conducted group teaching/learning interviews with 13 engineering students enrolled in second-semester calculus-based physics. We identified mathematics and physics resources activated by students and used the resource graph representation to describe students' coordination of various resources. The findings of this study provide further insights into students' difficulties with physics problems requiring integration.

Keywords: problem solving, mathematics, resources
PACS: 01.40.Fk

INTRODUCTION

Mathematical integration is widely used in many physics topics (e.g., electricity and magnetism). To fully understand and apply physics concepts, it is important for students to develop the skills of using integration in physics. A far more important goal for students is to invent their own integrals in new problem situations. Students from calculus are often good at evaluating a pre-determined integral, but they do poorly when setting up integrals in a physics problem [1,2]. Physicists use mathematics in a very different way than mathematicians do as the purpose of mathematics in physics is representing meaning about physical systems rather than expressing abstract relationships [3].

Many studies have investigated the difficulties students encounter with the use of integration in physics [1,4,5]. One of the major difficulties with setting up integrals involves interpretation of mathematical notations-differentials or infinitesimals (e.g., *dx*, *dr*, *dE*) [1,6]. We find that students interpret the meanings of differentials in various ways in different problem solving contexts. We explore students' difficulties from the manifold perspective, in which one's conception is determined by activation of resources depending on the context [7,8]. In this study, we are interested in the mathematics and physics resources students bring in with the use of differentials. Students often activate a variety of resources in a scenario and those resources are not isolated from each other. Thus, resources graphs [9] are used to show how the activation of one resource is linked to another and this representation provides a more complete picture of student reasoning in a given setting. Our work is an extension of Meredith & Marrongelle's work about the resources students used to cue integration [10]. They explored students' reasoning about why integration is needed, while we expand her work to explore students' resources used to set up infinitesimal equations -- a critical step for setting up integrals. We address the following research questions:

1. What mathematics and physics resources associated with differentials do students activate?
2. How do students coordinate their resources in a given problem scenario?

METHODOLOGY

We conducted group teaching/learning interviews [11] with 13 participants selected from a pool of 40 volunteers in a second-semester calculus-based physics course for engineers at a Midwestern university. All participants had taken pre-requisite calculus of single variables. Eight of these had previously taken and five were concurrently taking calculus of multi-variables.

Students were organized in five groups of two or three students each. The interviewer met with each group separately. During the interviews, students discussed problems together on a whiteboard. In total, eight 75-minute long interviews were completed over the semester. Each interview occurred within one week after students covered the related concepts in class. The problems discussed here are in the context of electricity & magnetism. Interviews were videotaped and coded for the analysis.

DATA ANALYSIS AND RESULTS

In viewing the videos we found several kinds of reasoning resources about differentials that students frequently use in different settings. We identified interesting episodes of students using these resources and transcribed them for detailed analysis.

Here we discuss three major resources that were prevalent in students' work: a "small piece/segment," a "point," and "differentiation." We provide examples of how students use these resources and draw resource graphs [9] to represent the network of students' resources used in a given situation.

"Small Piece/Segment"

Differential terms (e.g., dx, dr, dE) in physics equations often contain specific physical meaning based on physical systems. In an electric field problem (Fig. 1), to set up the equation for total electric field at point P, the traditional approach physicists would take consists of three major steps. The first step is to chop the whole rod into infinitesimal pieces each of length dx, carrying an infinitesimal charge dq. Then the second step is to set up the equation for dE (Equation 1), which is the infinitesimal electric field at P due to dq (Equation 2). Finally, the last step is to integrate dE to get total electric field E.

1. An insulated thin rod with length L has charge +Q uniformly distributed over the rod. Point P is located at a distance d from the right end of the rod. Find the electric field at point P due to this charged rod.

FIGURE 1. Electric field problem

$$dE = k\frac{dq}{r^2} \quad \text{and} \quad dq = \frac{Q}{L}dx \qquad (2)$$

The following transcripts are from a group of two students discussing how to find the equation for dq.

1 *Dave*: I guess we can do separating little
2 segments of dq.

They then set up the equation for dE (Equation 1) and dq (Equation 2). The interviewer ("I") comes over and asks them to explain both equations. Dave mainly explains the equation for dE (Equation 1) and Alice explains the equation for dq (Equation 2)

3 *Dave*: Well, since this is just the value that a
4 particular line segment is putting onto our P,
5 then that would only be a tiny segment of the
6 charge, which is what we described as *dq…*
7 *I*: Okay, can someone explain this equation? [points to equation 2]
8 *Alice*: Well, we have a charge *Q* over the entire
9 length of *L*, so this is just saying when you have
10 a little piece, cause you can write it differently,
11 you can write it as $\frac{dq}{dx} = \frac{Q}{L}$. So then it is just a ratio
12 of a whole charge over the whole length to a little
13 bit of charge over a little bit of length.

When describing the differential terms, both Dave and Alice frequently use the phrases "a line segment" or "a little bit of length" for dx and "a tiny segment of charge" or "a little bit of charge" for dq. It seems that students consider "*d*" to refer to a "small piece or segment" of a physical quantity in this context. In other words, they apply resources of a "small piece/segment" with their use of differentials. To illustrate how students coordinate resources to set up an equation for dq, a resources graph is drawn in Fig. 2.

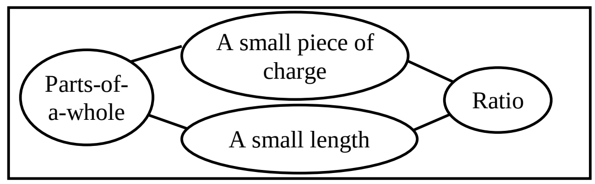

FIGURE 2. Resources graph for group 1

In lines 1 and 2, David starts with "separating little segments," and then sums up the effect due to each segment. He appears to be invoking "parts-of-a-whole" resources by interpreting the integral as a sum of many contributions [10,12]. Then students seem to activate a "small piece" resource for dq, and a "small length" resource for dx. Finally, they relate these by a ratio.

"Point"

When solving electric field problems, we find that students often interpret differentials dx and dq as point quantities. Physicists use "point" as an ideal physics model. When the size of an object can be neglected compared with the dimensions we are considering, we view the object as a point. It is by far the easiest and most commonly used model across many physics contexts. Thus, a "point" is a physics resource.

For the problem in Fig. 1, dq is the amount of charge carried by dx and it is acceptable to view dq as a point charge as it is extremely small. However, students' insufficient use of the "point" resource leads to their difficulties when setting up the equation for dq. Below, two students discuss the problem in Fig. 1.

14 *Aaron*: Well, we gonna to find like, would be
15 like summing up little charges at every point?
16 *Kelly*: Yeah, so it's dq, Q/L?

After this brief conversation, Kelly writes down an equation for dq (Equation 3), but later on, she adds dx on the right side of this equation when they start to set up an integral for total electric field. We suspect that students realize that something is missing on the right side as they need the variable of integration eventually. In the following conversation, the interviewer asks students to explain their thinking.

$$dq = \frac{Q}{L} \qquad (3)$$

17 *I*: Can you guys explain this equation? What is
18 the meaning of this equation?
19 *Aaron*: The charge at every single point is
20 charge divided by the distance.
Later, they continued
21 *Kelly*: Basically, the point charge is at each
22 point along L, is the total charge over its
23 length, so like what's it called?
24 *Aaron*: Charge density
Later, the interviewer prompted
25 *I*: So what does *dq* mean exactly?
26 *Aaron*: We knew we have to use integral to
27 sum everything up. We need to know what
28 we are summing up the whole time, so we
29 have to find…
30 *Kelly*: Just find the little charges by taking the
31 total charge over the length it's over, to find…
32 since it's uniform, we can find the charge
33 every point.

In lines 14 and 15, Aaron starts to talk about "summing up little charges" which we code as "parts-of-a-whole" resource for using integration. Then students activate the resources of "point charges" for setting up the infinitesimal equation *dq*. We notice that students keep using the terms "charge at a point" or "point charge" to describe *dq*. In fact, a "point" could have two different meanings: one represents a location in space and the other describes the characteristics of an object which is physically negligible. In the interview, students use "charge at a point" and "point charge" interchangeably and there is no evidence that they realize they two could mean different things. However, in this context, students talk about "summing up (the effect due to) little charges" (line 15). Thus, we suspect that it is more likely that students think of a "point" as representing an object with negligible physical size. In both lines 20 and 31, students think "charge at a point" is equal to the ratio "total charge over the length." If there were discrete charges distributed and each point carried the same amount of charge, the charge at each point is the total charge divided by the number of points. However, there is no discrete charge distribution. To relate the point charge with the ratio, students must consider the length of the rod as a proxy for the number of points on the rod. The resources graph (Fig. 3) depicts students' resources in this conversation.

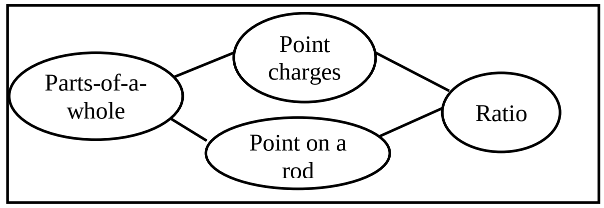

FIGURE 3. Resources graph for group 2

"Differentiation"

As students have solved many problems involving differentiation or integration in mathematics, they might have developed their own way of thinking about what differentials mean. During the interviews, some students tended to use differentials as a cue to mathematical operation such as differentiation or integration. In an example of a resistor problem (Fig. 4), students were reminded of the physics equation for resistance (Equation 4) which they learned previously in their lecture. To find the total resistance, the typical approach experts would use involves first chopping this whole cylinder into infinite numbers of small disks, finding the resistance of each infinitesimally thin disk *dR* (Equation 4), and integrating *dR*.

A material with length L and cross-sectional area A lies along the x-axis between x=0 and x=L. Its resistivity varies along the rod according to $\rho(x) = \rho_0 \cdot e^{-x/L}$. Find the total resistance of this cylinder between two end faces.

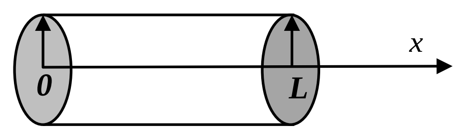

FIGURE 4. Resistor problem

Two students invent a wrong approach to set up an integral. They start with the basic resistance in equation (4), and plug in the resistivity function to get another equation (5). Then they find *dR* by taking the derivative with respect to x (Equation 5)

$$R = \frac{\rho L}{A} \quad \text{and} \quad dR = \frac{\rho(x)dx}{A} \qquad (4)$$

$$R = \frac{\rho(x)L}{A}, \text{ so } dR = d\left(\frac{\rho(x)L}{A}\right) = \frac{d\rho(x)}{A}L \qquad (5)$$

34 *I*: So why did you take the derivative of this?
35 *Zad*: Um, because we just plugged it into, we
36 plugged what *R* was, into the integral of *R*
37 basically. So we need to take the derivative of it.
38 So we can plug into *R*. Because we basically pull
39 *dx* out of nowhere, because the derivative of the
40 only changing function, then we require *dx*, we
41 need to integrate that.
Later, they continued
42 *Zad*: We have the function, you have to take the

derivative so you can take the integral. I don't know how to explain it other than mathematically.
Alan: Just, you need to take the integral across the whole thing. So in order to do that, we have to do a derivative, but it's basically just taking the difference from one to the other.

In lines 40 and 42, by talking about "changing function" or "function," Zad is focusing on the resistivity of the cylinder which is a function of x. We also have evidence from an earlier conversation that Zad uses the function as a cue for integration. Thus, we recognize that Zad activates the symbolic form of "dependence" which is described in Sherin's work as "a whole depends on a particular symbol that appears in the expression" [12]. The dependence resource was identified as a cue for using integration in earlier work [1,10]. Upon realizing that integration is needed, Zad applied the resources of "taking derivative" when setting up *dR*. The resource graph is shown in Fig. 5.

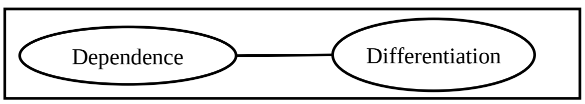

FIGURE 5. Resources graph for group 3

SUMMARY

We identified three resources students used while applying differentials in physics integration problems. A "small piece/segment" resource refers to a small portion of a physical object. In the electric field problem, upon activating a "small segment" resource, students in group 1 successfully related *dx* with length and *dq* with charge of a small line segment. They correctly set up the expression for *dq* which relates a small piece of charge with a small piece of length *dx*.

A "point" resource is typically used when the physical size of an object can be neglected. In the line of charge problem, students in group 2 view the continuously distributed line charge as a collection of point charges. In order to find the amount of charge at each point, they used the total charge divided by total length of the rod. It is plausible that students consider the length of the rod as a proxy for the number of points in order to set up a seemingly reasonable expression for *dq*. It is appropriate to use a "point" resource, but it is insufficiently used by students. The "differentiation" resource indicates that students consider "*d*" to be a mathematical operator without concrete physical meaning that they "basically pull out of nowhere." Activation of this resource leads students to invent an approach which they do not know how to explain "other than mathematically."

Appropriate scaffolding must be provided when students activate a resource or link resources in ways that are unproductive. When activating a "point" resource, possible strategies might include asking students to perform a unit analysis or a comparison between discrete and continuous charge distribution. For students activating a "differentiation" resource, it might be helpful to guide them through a qualitative analysis of why and how integration is used before they begin to use mathematical symbols and equations.

This article reports on three conceptual resources related to differentials. However, there might be other resources which are not identified given that the student population is small. Due to the limits of paper length, we only present examples of student work in two physics contexts. Within the three resources, some resources strongly rely on the context but others are widely used in many contexts. For instance, a "point" resource is used by some students in contexts involving a straight or curved line (e.g., a line or arch of charge); whereas a "small piece/segment" resource is used by students in various physics contexts (e.g., potential integral, Ampere's law).

ACKNOWLEDGMENTS

This work is supported in part by the National Science Foundation grant 0816207. Opinions expressed are of the authors and not of the Foundation.

REFERENCES

1. A. Orton, *Educational Studies in Mathematics*, **14**(1), 1-18 (1983).
2. D. Nguyen and N. S. Rebello, *Phys. Rev. ST Phys. Educ. Res.* **7**, 010113 (2011).
3. E. F. Redish, "Problem Solving and the Use of Math in Physics Courses," presented at the *World View on Physics Education in 2005:* Delhi, August 21-22 (2005).
4. F. R. Yeatts and J. R. Hundhausen, *Am. J. Physics* **60**, 716 (1992).
5. L. Cui, N.S. Rebello, P. R Fletcher and A.G. Bennett, "Transfer of Learning from College Calculus to Physics Courses," in *2006 National Association for Research in Science Teaching Conference CD*.
6. J. Von Korff and N. S. Rebello, *Phys. Rev. ST Phys. Educ. Res*. **8**, 010125 (2012).
7. A. A. diSessa, *Cogn. Instr*. **10**, 105-225 (1993).
8. D. Hammer, *Am. J. Physics, PER Suppl.*, **68**:7, S52-S59 (2000).
9. M. C. Wittmann, *Phys. Rev. ST Phys. Educ. Res.* **2**, 020105 (2006).
10. D. C. Meredith and K. A. Marrongelle, Am. J. Physics **76** (6), 570 – 578 (2008).
11. P. V. Engelhardt, E. G. Corpuz, D. J. Ozimek, and N. S. Rebello, "The Teaching Experiment - What it Is and What it Isn't," in *2003 Physics Education Research Conference*, edited by J. Marx, K. Cummings, and S. Franklin, AIP Conf. Proc. 720, Melville, New York, 2004, pp. 157-160.
12. B. L. Sherin, Cogn. Instr. **19**, 479-541 (2001).