Investigating students' mental models and knowledge construction of microscopic friction. II. Implications for curriculum design and development

Edgar D. Corpuz[1] and N. Sanjay Rebello[2]

[1]*Department of Physics and Geology, University of Texas-Pan American, 1201 West University Drive, Edinburg, Texas 78539, USA*

[2]*Department of Physics, 116 Cardwell Hall, Kansas State University, Manhattan, Kansas 66506-2601, USA*

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Our previous research showed that students' mental models of friction at the atomic level are significantly influenced by their macroscopic ideas. For most students, friction is due to the meshing of bumps and valleys and rubbing of atoms. The aforementioned results motivated us to further investigate how students can be helped to improve their present models of microscopic friction. Teaching interviews were conducted to study the dynamics of their model construction as they interacted with the interviewer, the scaffolding activities, and/or with each other. In this paper, we present the different scaffolding activities and the variation in the ideas that students generated as they did the hands-on and minds-on scaffolding activities. Results imply that through a series of carefully designed scaffolding activities, it is possible to facilitate the refinement of students' ideas of microscopic friction.

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I. INTRODUCTION

Our research has shown that most students hold onto the idea that friction at the atomic level is simply due to mechanical interactions [1]. By mechanical interactions we mean interactions similar to those observed in the macroscopic world, such as between two billiard balls colliding. While physicists may infer that these interactions are in fact electrical in nature, students often consider mechanical and electrical interactions to be qualitatively different. The students' view of friction at the atomic scale attributed to what they might call mechanical interactions is evident from the models they used in explaining friction in different contexts (e.g., friction between two wood surfaces, friction between wood and sandpaper surfaces); in explaining why static friction is greater than kinetic friction; and in explaining the lubricating mechanism of oil. When students were asked to sketch how the smoothest surface would look at the atomic level, they often drew atoms lining up. When asked if there was still friction when two such surfaces come into contact and move past one another, we often heard students say, "There will still be friction because there is still some contour in them (atoms)." Only one student (out of 11) consistently used electrical interactions in explaining friction. Thus, for most students, what they view as mechanical interactions of the macroscopic world are also applicable to explain friction at the microscopic scale.

The aforementioned findings motivated us to do further research on how students can be helped to improve their present models of friction. In this context, we decided to conduct teaching interviews with introductory physics students with the aim of studying the dynamics of their model construction as they interact with scaffolding activities. Specifically, we tried to address the following research questions.

(1) What scaffolding can facilitate students to reorganize and reconstruct their models of microscopic friction?
(2) What are the variations in students' interactions as they progress through a series of model-building activities?
(3) What teaching interventions and/or instructional strategies can be designed to help students come up with a more scientifically accepted model of microscopic friction and to what extent are these strategies successful?

The study of microscopic friction is an emerging area, and unlike other areas of physics that are typically addressed in an undergraduate physics curriculum, these ideas are still being developed by experts. In the next section, we present a contemporary model of how friction varies with surface roughness. We will also present experts' views about this emerging area of research.

According to Rabinowicz [2], friction varies with surface roughness approximately as shown in Fig. 1. It can be seen that friction goes up when the roughness increases due to interlocking of asperities. Friction also goes up when the surface roughness decreases due to the growth of contact area and adhesion between atoms.

Target ideas for students

Based on results of the clinical interviews with students, interviews with experts, and a literature survey,

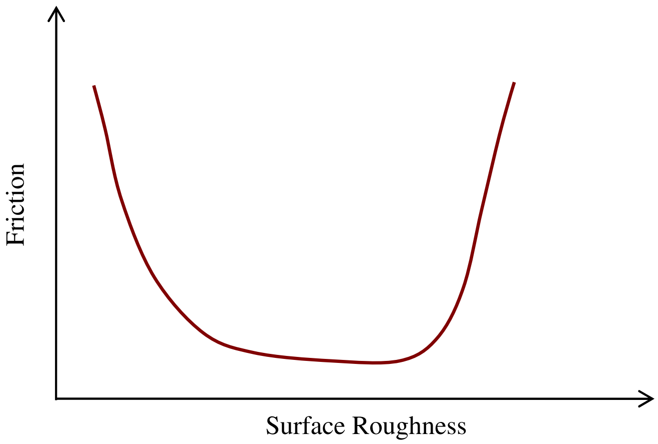

FIG. 1 (color online). Approximate graph of friction versus surface roughness.

the following aspects of microscopic friction were adopted as the target ideas that we aimed for in refining students' ideas of microscopic friction.

(1) Friction is due to electrical adhesion of atoms.
(2) Friction is dependent on the atomic contact area.
(3) Friction on atomically smooth surfaces is large.
(4) Friction varies with roughness as in Fig. 1.

II. METHODOLOGY

In investigating the dynamics of students' knowledge construction and reconstruction we used the teaching interview [3] as a research methodology. The philosophical basis for this methodology is consistent with the constructivist views of Piaget [4] as well as those of Vygotsky [5]. By stating that our philosophical basis is consistent with both of these views, we do not intend to imply that Piaget's and Vygotsky's views are similar. Rather, we imply that we draw from the ideas of both these authors in the design of our research methodology of the teaching interview. The teaching interview methodology also finds its underpinnings in the contemporary views of transfer of learning [6–8]. One of the aims of a teaching interview is to provide a bridge between educational research and teaching practice. Cobb and Steffe [9] assert that the interest of a researcher during the teaching experiment is in generating hypotheses on what a student might learn and finding ways and means of fostering learning in a given context.

The teaching interview is a mock instructional setting in which the teacher-researcher influences the knowledge construction process of students by providing pedagogically appropriate scaffolding. It provides a rich context in which one can study the dynamics of students' knowledge construction and reconstruction as they interact with a learning material, with other students, and with the teacher-interviewer. It is important to point out that it is not necessarily the goal of the teaching interview to find the optimal effective teaching methods or the best way to teach students. Rather, it is the goal of the teaching interview to investigate the variations in the trajectories of student learning and the factors that influence these trajectories. The research outcomes from the teaching interview can be used for planning learning experiences for students in helping them better understand a given phenomenon as well as for building our own theory or model of how students learn.

One of the main concerns of the teaching interview methodology arises due to the threat to the validity of data collected from the teaching interviews. If the student participant frames the process as one in which the teacher (researcher) expects the student to learn a concept that the teacher is intending to teach, the student might be more likely to provide information that they believe the teacher wants to hear and somehow attempt to mask their lack of understanding or true opinion about a subject. We minimize this threat to validity by emphasizing to the student that it is not our intent through this process to provide them a scientifically correct understanding of the phenomena. Rather, our goal is to explore how they think about phenomena and to probe how their thinking might or might not change in response to certain questions or information.

The initial set of questions for the teaching interview we conducted was primarily based on the series of questions asked during the clinical interview in the previous phase of the project. In the clinical interview the interviewer avoided, as much as possible, prompting the interviewee and changing his or her initial ideas during the interview. However, in the teaching interviews, questions were asked so that students were prompted to think in a certain way. The way the questions were phrased was modified in a way that they increasingly become leading questions. For example, in sketching surfaces, students were deliberately asked to sketch the surfaces at the atomic level.

The preliminary set of scaffolding activities included some of the model-eliciting activities used in the clinical interview [1]. It included the activity of sliding fingers across surfaces and sketching them at the atomic level, the dragging of a wooden block across wooden and sandpaper surfaces. Thus, the teaching interview was a natural outgrowth of the clinical interview conducted in the first phase of the research.

As the teaching interviews progressed through time, the sessions converged to a protocol that did not deviate significantly from what was used in the previous sessions. Some of the model-eliciting activities from the clinical interviews were adopted as part of the scaffolding activities for the teaching interviews. Below we present the different scaffolding activities that were used during the teaching interview as we attempted to refine students' ideas of microscopic friction.

A. Scaffolding activities

Activity no. 1: Feeling and sketching of rough and smooth surfaces.—In this activity students were asked to slide their fingers across a wooden block surface, the sandpaper surface, and the wooden plank surface. Students were then asked to sketch the different surfaces at various length scales down to the level where they presume to see the atoms. Major questions and prompts included the following.

Please feel the surfaces by rubbing your fingers across each.

How would you compare the surfaces?

Sketch what the surfaces would look like to you at the level where you see the atoms.

Activity no. 2: Dragging of wooden block across a wooden plank and sandpaper surface.—In this activity students were first asked to predict how the force needed to pull the wooden block across the two different surfaces would compare (Fig. 2). They were then asked to give reasons for their predictions before dragging the wooden block across the two different surfaces and describing their observations. Questions or prompts during this activity included the following.

If we pull the block on one end of the spring scale, could you please predict what happens to the reading [of the scale] as we move the block across the different surfaces (sandpaper and wooden plank)?

Why is it that we need a finite amount of force in order to start the block moving?

If we drag the block along the wooden plank and along the surface with sandpaper, could you please predict how the friction force would compare in the two situations?

Activity no. 3: Graphing the variation of friction with surface roughness of both surfaces.—Students were asked to sketch a graph showing how friction force varies with the surface roughness of pairs of sliding surfaces. They were also explicitly asked to explain the details of their graph. Questions or prompts included the following.

Please make a graph on how friction force would vary with the roughness of the sliding surfaces. Explain the details of your graph.

What happens to the friction force as the surfaces become rougher and rougher?

What happens to the friction force as the sliding surfaces become smoother and smoother?

Activity no. 4: Metal blocks activity.—The purpose of this activity was to challenge students' prior ideas about friction. In this activity, students explored friction between a very smooth pair of surfaces of metal blocks and a smooth-rough pair of surfaces of the same metal block (see Fig. 3). Students were first asked to compare the smoothness of the different surfaces of the metal block by letting them slide their fingernails across the different surfaces. They were then asked to give their predictions and reasons for their predictions of the pair of surfaces for which the friction would be greater. Students tended to predict that there would be more friction between the smooth-rough pair of surfaces because there would be more "interlocking." However, after doing the activity, they would later find out that it would be harder to slide the smooth-smooth pair of surfaces across each other rather than the smooth-rough pair of surfaces. This activity engaged students in cognitive conflict which they mostly resolved through other activities that followed. Through this activity students' current ideas were challenged. This put the students in a mode of considering alternative explanations of friction at the atomic level.

Questions and prompts included the following.

Slide your fingernail on the two surfaces. How does the surface roughness or smoothness compare?

Please sketch what the two surfaces would look like at the atomic level.

Predict how the friction force compares when we slide the smooth surface of the metal block on the other smooth

FIG. 2 (color online). Dragging of wooden block.

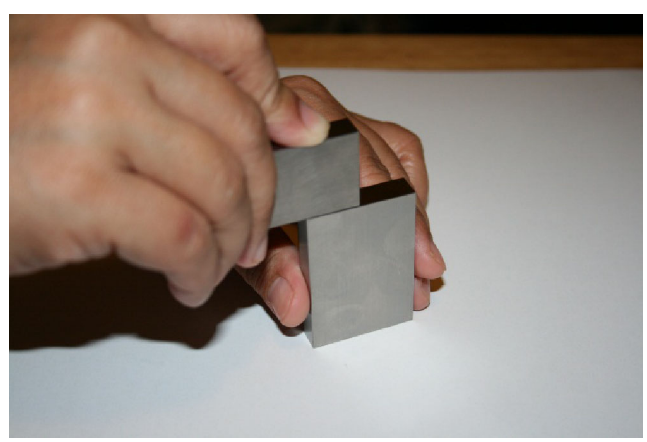

FIG. 3 (color online). Sliding metal gauge blocks over each other.

surface and the case where we slide the smooth on the rougher sides. Explain your prediction.

Now please explain your observations.

Activity no. 5: Paper-transparency activity.—Students were provided with a transparency and plain sheet of paper (see Fig. 4). They were asked to first rub the transparency with fur and then slide the sheet of paper across it. Students typically noticed that they needed to exert a force to pull a flat sheet of paper across the transparency rubbed with fur. Next they crumpled the same sheet into a ball and after straightening it out they were asked to pull the sheet across the same transparency. Most students noticed that it was much easier to pull the crumpled sheet of paper across the transparency compared to the flat sheet of paper.

This activity was used to resolve the cognitive conflict brought about by the metal gauge block activity. Through this activity students were provided clues to the role of electrical interactions and the real area of contact between two surfaces.

How do the apparent areas of the crumpled and uncrumpled paper compare?

Could you please predict how the friction force between the transparency and crumpled paper compare with the friction force between the transparency and the uncrumpled paper.

Why do you predict that?

Could you please explain your observation?

Activity no. 6: Relating the paper-transparency activity with the metal blocks activity.—Later in the teaching interview students were explicitly asked to relate their observations in the paper-transparency activity with the metal blocks activity. Through this activity, students came to realize that friction can be large when surfaces become smooth because the real area of contact increases. The real area of contact in this case is not necessarily the visible area of the surface. Rather it is the sum total of the area of the individual atoms contacting each other. Questions and prompts used include the following.

Could you please relate what you have just observed here with that of what is happening with the metal blocks.

How does the number of points of atomic contact compare in the two situations?

Activity no 7: Revisiting the friction versus roughness graph.—In light of the new phenomenon explored in activity no. 6, students were explicitly directed to go back and make sense of their previous prediction of how friction varied with surface roughness. The purpose of this was to make students reflect on their initial graph and realize that their initial graph was not necessarily consistent in light of the activities that they had just completed.

Also, in this activity the students were explicitly provided the opportunity to revise their initial model of how friction varies with the surface roughness and represent this model on the graph of friction versus surface roughness that they had revisited in the previous activity. Questions and prompts included the following.

Do you still go with your previous graph? If not, how would you modify it?

Explain the details of your new graph.

What happens to the friction force as the surfaces become rougher and rougher?

What happens to the friction force as the sliding surfaces become smoother and smoother?

B. Participants of the study

A total of 18 students enrolled in introductory physics courses participated in the individual teaching interviews. Table I shows the interviewees as per the physics course they were enrolled in during the conduct of the research. As can be seen from the table, two students were enrolled in Physical World, eight were enrolled in General Physics, and eight were enrolled in Engineering Physics. The Physical World course deals with the conceptual treatment of different physical science topics and is typically taken by nonscience major students. The General Physics course is an algebra-based course dealing with mechanics, heat, fluids, oscillations, waves, sound, electricity and magnetism, light and optics, and atomic and nuclear physics typically taken by life science majors. Meanwhile,

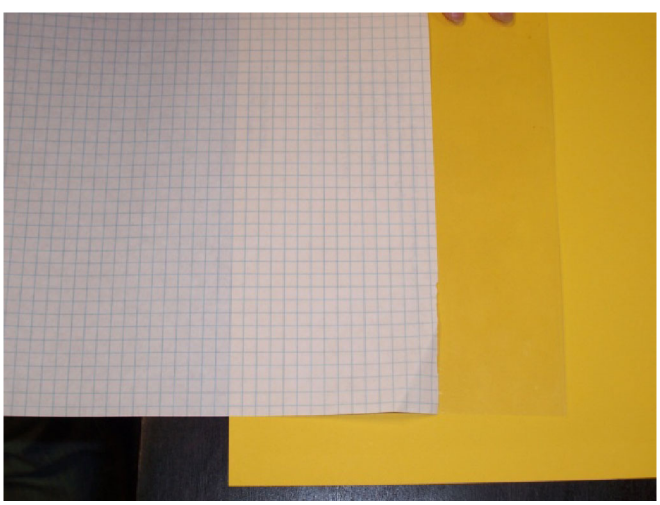

FIG. 4 (color online). Paper over a transparency.

TABLE I. The interviewees as per their physics course enrolled.

Physics course	No. of students
Physical World	2
General Physics I	4
General Physics II	4
Engineering Physics I	4
Engineering Physics II	4
Total	18

Engineering Physics is a calculus-based physics course which deals with mechanics, heat, sound, electricity, magnetism, light, and modern physics typically taken by physics and engineering majors.

III. RESULTS AND DISCUSSION

The goal of this phase of the research was to determine how students dynamically construct their ideas to describe microscopic friction in the context of a teaching interview. We used a phenomenographic approach [10] to analyze data. Significant statements were extracted from the transcripts and were the focus of the analysis. These significant statements were analyzed using the two-level transfer framework [11] as a guide to determine the associations or ‘‘relations of similarity’’ [8] that students organically create as they reason through the activities. This process was validated by two researchers and discussed until a consensus was reached. Each level of association was then treated as a category. We kept track of associations that the student made in different segments of the interview and determined how the student used these associations to construct a model of friction at the microscopic level. We present the results of the individual teaching interviews in terms of the variations of the associations that students make per scaffolding activity as described in the methodology section.

Activity no. 1: Feeling and sketching of rough and smooth surfaces.—In activity no. 1, students were asked to feel and sketch both rough (sandpaper) and smooth (wooden block) surfaces and sketch the surface at the atomic scale. Students made four different types of associations. A vast majority (17 out of 18) of the students that were interviewed associated smooth surfaces with atoms lining up [Fig. 5(a)] and with rough surfaces having atoms arranged in an up and down manner [Fig. 5(b)]. Two students (one calculus-based physics and one algebra-based physics student) associated smooth surfaces with atoms closer together while one of them (a calculus-based physics student) appeared to think otherwise. Yet another student (algebra-based physics student) associated surface roughness with sizes of atoms.

Activity no. 2: Dragging of wooden block across a wooden plank and sandpaper surface.—In activity no. 2 students were asked to slowly drag a wooden block across a smooth wooden surface as well as a rough sandpaper surface. In explaining the cause of friction when the wooden block is dragged across the sandpaper surface, 10 out of 18 students used ‘‘the catching of ridges’’ explanation. These students associate friction with the catching of the ridges of the surfaces. According to one of these students:

‘‘... the ridges are catching on each other. That's how I would describe it. And in here (wood block over the plank), there's not much ridge on the smoother surface to catch on so I guess that's how I will describe it. It's just the catching of the ridges.’’

One student associated more friction with rough surfaces (e.g., sandpaper) because of the atoms fitting real tight. According to this student:

‘‘... they're gonna fit real tight together and you are trying to move one that way and the other that way. It will take a lot of force...’’

Meanwhile two students (calculus-based physics students) associated more friction with the wooden block on the sandpaper because of the surfaces grabbing. According to one of the students:

‘‘... on here (block over the sandpaper) there's edges that actually grab on to the block and create the friction.’’

One of the algebra-based students explained the friction between the surfaces as follows:

‘‘Because that is not that smooth, so they will not move as easily. There's gonna be more interaction among the molecules ...There will be more interaction because it is more jagged. They (atoms) would have the chance to get closer. They would get closer to each other.’’

This student associated more friction with the wooden block on the sandpaper because there's more interaction and more atoms getting closer.

Activity no. 3: Graphing the variation of friction with surface roughness of both surfaces.—In activity no. 3, students were asked to sketch a graph of friction versus surface roughness. There were three variations on the graph that students generated when they were asked to graph the friction force versus roughness of the sliding surfaces. Fifteen of the 18 students associated increasing roughness and smoothness with increasing friction. All of these 15 students also pointed out that the relationship between roughness and friction is linear (see Fig. 6). One of the 18 students associated increasing friction with increasing roughness but remarked that the relationship was nonlinear. It is interesting to note that one of the students (calculus-based physics student) believed that friction force increases to some point as the roughness increases and it then decreases with increasing roughness. The student believed that as you increase the roughness the friction force will increase because the rougher surface is getting ‘‘more grip’’ but at some point the gripping decreases because of reduced surface area making contact. According to this student

‘‘... here it (friction force) is increasing because your rougher surface is getting more grip. But at some point

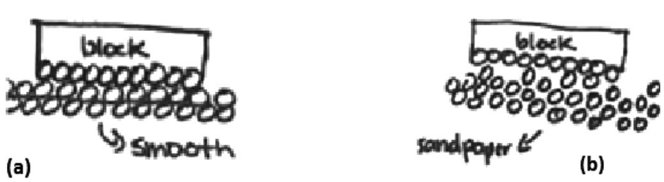

FIG. 5. (a). Smooth surface (atoms lining up). (b) Rough surface (up and down arrangement of atoms).

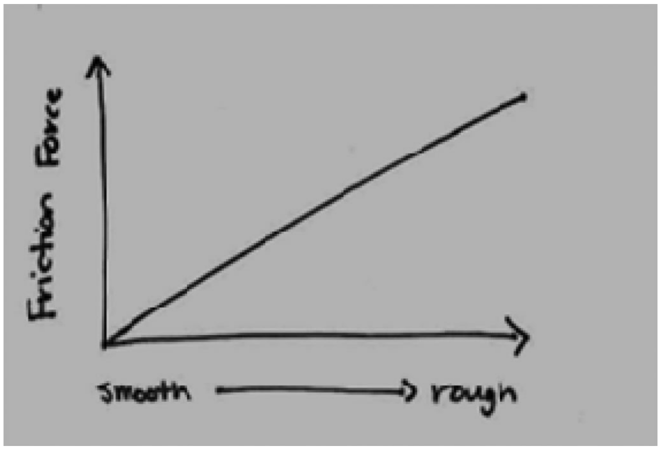

FIG. 6. Typical student sketch of friction versus roughness.

your rougher surface will no longer get more grip by digging into it. It also reduces the surface area making contact. Cause at some point it will look like you'll have a series of points.''

Activity no. 4: Metal blocks activity.—In activity no. 4 the students were asked to predict which pairs of the metal block surfaces (smooth on smooth versus smooth on rough) would be easier to slide over each other. Seventeen of the students predicted that there will be more friction in the case of the smooth on rough. For most students rougher means more friction. According to one of the students there will be more friction on the smooth and smooth because of the tendency of the atoms to bond together. The reason that the student made such a prediction is that he previously learned this in his engineering materials course.

After sliding the pairs of surfaces together, students found that it is actually harder to slide the smooth sides together. This activity was designed to create cognitive conflict or disequilibrium in the minds of students in that their predictions are different from their observations. When asked to explain their observations, six of the students could not say anything about what was happening. The other students tried to resolve the conflict themselves by using explanations based on previous experiences with magnets and previous explanations in the teaching interview. For those students who attempted to explain the phenomena, their explanations were one of the following.

''Surfaces are quite similar, so they stick together.''

''Smooth on smooth are like magnets which attract each other.''

''More atoms touch on smooth on smooth, because the smooth sides are curved.''

''Atoms line up in the smooth on smooth case, so the atoms weld together.''

After doing further explorations with other metals, students with explanations that the surfaces are magnets later abandoned their explanations and were unable to give further explanations of what was happening.

Activity no. 5: Paper-transparency activity.—The papers and transparency activity was designed to help students resolve the above cognitive conflict and help them construct more plausible explanations for their observations in the metal blocks activity (activity no. 4). A majority of the students predicted correctly that it will take more force to pull the flat sheet of paper across the transparency. The reasoning resources included the following.

''There would be more static on the flat sheet of paper.''

''The uncrumpled paper and the transparency are both flat and thus can connect more.''

''There will be more area touching.''

''They would behave in the same manner as the smooth on smooth metal surfaces.''

Two of the students (calculus-based physics students) predicted that there would be the same friction in the crumpled and uncrumpled cases; the reason is that they had learned in class that friction does not depend on the area. According to these students friction was simply given by the equation $f = \mu N$.

The papers and transparency activity helped students recognize the role of the area of contact on the friction force between two surfaces. Most of the students (15 out of 18) explained that in the uncrumpled paper on the transparency there was more area touching, which is the reason why there is greater force to pull and hence greater friction between the surfaces. Students with more detailed explanations talked about more charges involved, more atoms touching, and more static effect on the surfaces that have more contact.

Activity no. 6: Relating the paper-transparency activity with the metal blocks activity.—When asked to relate what they had done with the papers and transparency with what they observed with the metal blocks, students used the following reasoning resources.

''More atoms touching on the smooth on smooth, so there will be more attraction.''

''More surface area touching, so there will be more charges involved on smooth on smooth sides.''

''More molecules making contact.''

''More charges involved.''

At this stage of the teaching interview, students are now making the associations at the microscopic level: more friction with more charges interacting and more friction with greater area of contact.

Activity no. 7: Revisiting the friction versus roughness graph.—

When students were asked if they would still go with their previous graph of friction force versus roughness of the surfaces, most of the students would respond in the negative. They would then modify their graph, and a majority of the students (17 out 18) automatically drew a graph similar to the U-shaped graph in Fig. 7.

On the left-hand side of the graph students usually talked about the area of contact between charges as the primary

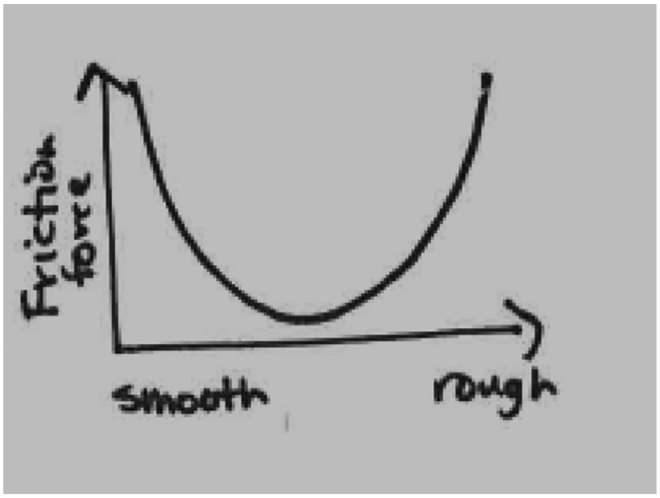

FIG. 7. Modified graph of friction force versus roughness of the sliding surfaces.

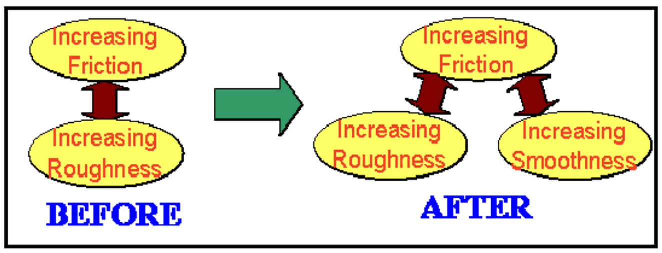

FIG. 8 (color online). Knowledge construction via incremental change process.

reason why the friction becomes high when surfaces become so smooth. When the sliding surfaces are very rough (right-hand side of the graph), the bumps on one surface might get caught in the valleys on the other surface also causing high friction. In coming up with the graph, students were engaged in a process of incremental change as per Wittmann [12]. Initially students associated increasing roughness with increasing friction. They then assimilated the association between increasing smoothness and increasing friction (see Fig. 8). However, in providing a more detailed description of the modified graph, it appeared that students needed to make dual construction [12]; that is, they activated two different associations: one association for very smooth surfaces and another association for very rough surfaces.

IV. CONCLUSION

Our results show that a vast majority of the students associate rough surfaces with a zigzag arrangement of atoms and a smooth surface with atoms lining up. At the beginning of the teaching interview, most students typically will start out associating increasing roughness with increasing friction and increasing smoothness with decreasing friction. The scaffolding activities build on these associations in order to guide the refinement of students' ideas of friction at the microscopic level. Through the metal blocks activity students are put into a state of cognitive conflict in that their prior associations with increasing roughness with increasing friction fail to explain their observation that it was harder to move the smooth on smooth metal block surfaces compared to the smooth on rough metal block surfaces. The papers and transparency activity helped them resolve this conflict by making students realize the role of the area of contact and charges in explaining friction. The papers and transparency and metal blocks activities seemed to facilitate the refinement of students' ideas of friction at the atomic level. The series of hands-on and minds-on scaffolding activities used in the teaching interview seemed effective in making students construct the U-shaped graph of friction as depicted in Fig. 1.

In constructing their models of association regarding microscopic friction, the analysis suggests that students undergo the processes of incorporation and displacement [13]. We have also observed the processes of incremental change and dual construction [12] occurring as students construct and reconstruct their ideas about microscopic friction. Although these students have different physics backgrounds, they have the necessary internal knowledge to activate the appropriate associations with respect to the established target ideas of microscopic friction, and for the major part no differences were observed between students with different backgrounds.

In addition to the aforementioned conclusions, this research has demonstrated that the teaching interview can serve as a useful step in the design of curriculum materials. By elucidating the fine-grained detail of students' knowledge construction processes, the teaching interview enables the researchers to create appropriate scaffolding activities that can facilitate learning along a desired conceptual trajectory. Thus, the broader impact of this research is that it has informed the process by which science educators and researchers can develop effective curricular materials to foster conceptual change.

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