Facilitating Students' Problem Solving Across Representations in Introductory Physics

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Abstract

We report on a study to investigate the common difficulties that students encountered when solving problems in different representational forms in an introductory physics course and the scaffolding that might help students overcome those difficulties. We conducted teaching/learning interviews with 20 students in a first semester calculus-based physics course. In those interviews, students had a chance to work with problems sharing the same physical similarities but different in representation. Students were provided verbal hints whenever they were unable to proceed. We present the interview problems, common difficulties that students had and the hints we provided to help them overcome those difficulties.

Introduction

The use of multiple representations in solving introductory physics problems has been of great interest to physics education researchers. There have been studies addressing the benefits of using different representations in solving physics problems (De Leone & Gire, 2005; Van Heuvelen & Maloney, 1999), the strategies to facilitate students' problem solving across representations (Van Heuvelen & Zou, 2001), as well as other pedagogical aspects of helping students construct representations. (Dufresne, Gerace, & Leonard, 1997; Heller & Reif, 1984; Kohl, Rosengrant, & Finkelstein, 2006; Larkin & Simon, 1987; Rosengrant, Van Heuvelen, & Etkina, 2006; van Someren, Reimann, Boshuizen, & de Jong, 1998). Meltzer (2005) found that students' performance on similar problems posed in different representations might yield significantly different results.

However, there have not been many studies addressing the specific types of difficulties students might have with different kinds of representations as well as the difficulties they might encounter when transferring their problem solving skills across representations. Rosengrant, Van Heuvelen, & Etkina (2009) investigated the thought processes of students when using particular representations, but did not figure out the difficulties students had with each representation. Tuminaro (2004) addressed the difficulties students had with mathematics in physics, but did not address the difficulties students might have with mathematical representations of physics problems.

In this study, we investigated the difficulties students encountered when solving introductory mechanics problems in numerical, graphical, and equation representations. Specifically, we addressed the following research questions:

(i) What are the common difficulties that students encounter when solving physics problems in different representations?

(ii) What kinds of hints are useful in helping students overcome those difficulties?

This study is the first step toward the building of an online system which is responsive to students’ need for assistance when they are unable to proceed in a problem. The common difficulties and successful hints determined from this study constitute a research-based database for that online system, which may address students’ needs better than the current systems built on the intuition of the creators on what students need help with and what prompts are helpful. So in this study, we focus on the mechanics of solving the problems and we are interested in the specific kinds of difficulty students encountered and corresponding hints that might help students overcome those difficulties.

Theoretical Framework

The theoretical framework that is applicable to this study is Vygotsky’s Zone of Proximal Development (ZPD) (1978). The ZPD is defined as the difference between what a learner can accomplish by her/himself and what they can accomplish with help from another more experienced individual. The help provided by the more experienced peer or instructor is known as scaffolding. Eventually, the learner develops the necessary knowledge and skills to a point where they are able to accomplish the task without help from others.

In this study, students were provided with a previously unseen problem and asked to solve it. As they progressed through the solution, they encountered difficulties. The causes of these difficulties ranged from lack of attention to certain features of the problem to deeper mathematical or conceptual difficulties. To enable students to overcome these difficulties, they were provided with explicit or implicit hints that were mostly in the form of questions that were designed to get the students to change the way in which they were thinking about the problem in both minor as well as significant ways.

In terms of Vygotsky's notion of the ZPD, the difficulties encountered by students represented the lower boundary of the ZPD, i.e. the point at which they were was unable to make progress on the problem themselves and needed assistance from the interviewer to proceed. The hints provided by the interviewer to enable the student to overcome the difficulties were the scaffolding. The interviewer was expected to provide hints that were within the learners' ZPD. A hint that was beyond the ZPD was not understood by the learner and hence would not facilitate the learner to make progress toward finding a path to the solution. Thus we can use Vygotsky's theory of ZPD to interpret the difficulties and hints provided to the learner to enable them to solve the problem themselves.

Methodology

Teaching Interviews

The teaching/learning interview is a research tool that we have adapted from the Teaching Experiment, which has been used effectively by many researchers to gain insights into the strategies that support student learning (Steffe, 1983; Steffe & Thompson, 2000). Our adaptation of a teaching experiment, which we call the teaching interview, was developed by Engelhardt, Corpuz, Ozimek, & Rebello (2003). The teaching interview differs from clinical interview in an important way. While the goal of a clinical interview is typically to investigate students' ideas without influencing these ideas, the goal of a teaching interview is to specifically facilitate students' development of ideas and to provide the necessary scaffolding. In a teaching/learning interview the researcher can probe how a learner reacts to different kinds of scaffolding. In this study, teaching/learning interviews were designed to investigate as well as to scaffold students' problem solving processes. Specifically, we were interested in investigating how learners apply mathematical concepts to physics problems and how they can transfer their problem solving skills across different problem representations.

Participants

We conducted individual teaching/learning interviews with 20 students randomly selected from a pool of 102 volunteers enrolling in a first-semester calculus-based physics course (covering mechanics and thermodynamics at the introductory level). The majors of the students that participated in our interviews are given in Table 1.

Interview Details

Each student was interviewed four times during the semester, each time after an exam in their physics class. In each interview, students were asked to solve three problems, including a problem from the most recent exam in the course (original problem), a modified version of the original problem in which part of the information was provided as a graph (graphical problem) and a modified version of the original problem in which part of the information was provided as an algebraic equation (equation problem). All of the interview problems used in this study are presented in the appendix at the end of this paper.

Students were asked to think aloud as they solved the problems on paper. Verbal hints were provided by the interviewer whenever students were unable to proceed or were on the wrong track. All of the interviews were videotaped and audio recorded and students' worksheets were collected.

Data Analysis

For the purpose of identifying the difficulties and hints in the interviews, we created a "pseudo-transcript" of each interview in which we recorded the main ideas of each sentence in the conversation between the interviewer and the student and the writing of students on their worksheets. We deemed the pseudo-transcript to be sufficient for the purposes of our analysis, since it provides a clear description of the difficulties that students encountered while solving the problem, the hints that were provided to them to overcome the difficulty as well as their responses to the hints.

A difficulty was determined when the student explicitly (by saying sentences like "I don't know how to do this") or implicitly (by pausing for a long while -- more than 15 - 20 seconds) expressed an inability in accomplishing a specific task which contributed to the solution of the problem (e.g. identifying the principle to use, extracting information from a graph, setting up a correct integral, etc.) or revealed an error in implementing calculations (e.g. using incorrect value of quantities in an equation, using incorrect mathematical tools, etc.). The following conversation from an interview with a student on the equation problem of interview 3 demonstrates how the difficulties and hints were counted.

Interviewer: *Now that you know you have to find work done by friction, how would you do that?*

Student: *Could I plug in … zero for x at the beginning and 5 at the end?* [Difficulty on Quantity – does not know how to find work done by friction]

Interviewer: *In this problem, is the force constant?* [Hint on Information – suggest student take a more careful look at the information given]

Student: *No.*

Interviewer: *Then at each point you have a different value of F, so you don't know which value to plug into F.d. What should you do with a changing force?*

Student: *[pause] I know D=5m but I don't know how to find the F. It gives it, but I don't know what to do with F.* [Difficulty on Quantity – does not know how to find work from the equation of force as a function of displacement]

Interviewer: *For a constant force, work is F.d. How can you modify that to find work done by changing force?*

Student: *I don't know.*

Interviewer: *Okay. Suppose on a very short distance between two close points the force is really close to a constant that you can apply Fd. Then you do the same for the next points and add up everything. Then what do you have?* [Hint on Mathematical Meaning – helps student recognize the meaning of a mathematical process]

Student: *Work over the whole distance*.

Interviewer: *Right. And what is the mathematical operation for that process?* [Hint on Mathematical Process – triggers student to relate a mathematical operation and a mathematical process]

Student: *It's the summation but would that just be the integral?*

Interviewer*: Right.*

A hint was a question or a piece of information provided by the interviewer when the student encountered a difficulty when the student was on the wrong track toward the solution, or was unable to proceed. In contrast, a follow-up question was a question asked by the interviewer to clarify a student's idea and reasoning when the student was on the right track toward the solution. The following conversation from the beginning of the equation problem in interview 2 provides an example on the difference between a hint and a follow-up question.

Interviewer: *How is this problem different from the previous problems?* [Follow-up Question to start the problem]

Student: *This is a lot like the last problem. You're given a function instead of a graph and h will be found the same way. The only thing we have to find now is k.* [plug x = .2m into F(x) and get F = 80N] [This does not count as a difficulty yet because we are still not sure what the student intends to do]

Interviewer: *How does that force help you in finding k?* [Follow-up Question to clarify what student is doing]

Student: *Then I divide this force by the compression to get k.* [take 80N divide by 0.2m and get 400 N/m] [This is difficult because now it's clear that the student is misusing the equation of force as a function of displacement]

Interviewer: *Does the function of force look like F = kx?* [Hint on Information – suggests student looks at the force function in the problem statement]

Student: *No. So is it* $F = kx + kx^2$ *then?*

Interviewer: *No. This is a non-linear spring and it doesn't have a spring constant.* [Hint on Conceptual Information – provides student conceptual knowledge of physics]

A table of codes was developed to categorize the difficulties and hints. A code was assigned to each difficulty a student encountered and each hint the interviewer gave. The inter-rater reliability for coding was about 80% before discussion and about 99% after discussion between coders. The codes were then sorted into categories of difficulties and categories of hints. We determined the number of difficulties in each category a student encountered and the number of hints the interviewer provided in each interview and across many interviews. This analysis provided information on the major types of difficulties students encountered in solving a problem in a certain representation and how difficult a problem in this representation was in comparison to a similar problem posed in another representation. It also suggested the kinds and amount of help students needed to solve a problem. A qualitative analysis of the interview data also provided important insight into students' problem solving process, so it was also employed to determine emergent themes in students' performance. The emergent themes are described in detail in the following section.

Findings

We present the categories of difficulties students encountered, the strategies we used to facilitate them to overcome those difficulties, the effect of the sequence of problems on students' difficulties and the emergent themes in students' performance.

Categories of Difficulties

A total of 17 codes for students' difficulties were collapsed into eight categories.

- D-PRINCIPLE: Using the inappropriate principles to solve the problems or writing incorrect expressions of those principles. An example of this type of difficulty is when a student included only the work done by friction in 'Work-Kinetic Energy' theorem and not work done by other forces.
- D-QUANTITY: Using inappropriate physical quantities to describe the situation, confusing physical quantities, not knowing how to calculate a quantity or using incorrect units of physical quantities.
- D-FORMULA: Misinterpreting the meaning of a formula or not knowing the formulae for physical quantities such as rotational inertia of a sphere or potential energy of a spring.

- D-VALUE: Using inappropriate values of quantities to put in a formula or an expression. For example, using the incline length as height when calculating gravitational potential energy.
- D-GRAPH: Inability to read off values, extract and process information from a graph such as the vertical intercept or using the graph to calculate a physical quantity.
- D-MATH: Inappropriate use of mathematical concepts or incorrect manipulation of mathematical processes, e.g. confusing sine and cosine, incorrect setup or computing of integral.
- D-FUNCTION: Misinterpreting or not knowing how to use the equation provided to find the desired quantities. For instance, students plugged values of displacement into the equation of force as a function of displacement to find work rather than integrating the function.
- D-CALC: Simple mathematical errors in calculation such as not squaring velocity in calculating kinetic energy.

Categories for Hints

A total of 20 codes for the hints provided to students were collapsed into seven categories. Almost 80% of the hints were provided in the form of questions rather than direct statements to the students.

- H-PRINCPLE: Questions cuing students to reflect on whether a principle was applicable. For instance, "Is there any kind of friction in this problem?" was asked when students were not sure if the principle of conservation of energy applied.

- H-INFO: Asked students to look more carefully at the problem statement so that they could attend to previously overlooked information. For example, "What is the shape of the object in this problem?"
- H-QUANTITY: Students were asked to decide which physical quantity was applicable in a situation, to plan a strategy to find the desired quantity or to recognize the units of quantity.
- H-FORMULA: Helping students recognize the meaning of a formula, expression, equation, result, or recognize a mistake in their equations or formulas. For example, when students wrote the expression for conservation of energy with friction as $\Delta K + \Delta U = 0$, they would be asked, "How does friction affect your expression of conservation of energy?" which helped students recognize that the correct expression should be $\Delta K + \Delta U = W$, where W was the work done by friction.
- H-GRAPH: A direct question asking students to read information from the graph, such as, "At x = 0.2m, what is the force?" or an indirect question prompting them to use the graph to accomplish a task such as, "How do you calculate work using the graph of force vs. distance?"
- H-MATH: Reminding students of mathematical concepts they might have forgotten or confused, such as "Sine is opposite/hypotenuse." or helping them recognize the meaning of integral to set up the correct integral.
- H-CALC: Helping students recognize calculation errors by asking whether their results "made sense."

Besides the common difficulties students experienced while solving the problems in general, we were specifically interested in those difficulties caused by the change in representation. An "R" was added in front of a code each time it appeared to indicate a difficulty caused by the change in representation of the problems. That is, a difficulty a student encountered when doing a task which he/she had already done successfully in another representation within the same interview. For example, the code "D-QUAN-CALC" was assigned when a student did not know how to find potential energy stored in a spring in the original problem, while the code "R-D-QUAN-CALC" was assigned if a student did not know how to do that from the given graph or equation of force as a function of spring compression in the same interview. A similar coding strategy was also applied for the hints.

Sequencing Effect

We also investigated the effect of the sequence of problems presented to students on their performance by giving half of the students the graphical problem before the equation problem (which we called the G-E sequence) and the other half of the students the equation problem before the graphical problem (which we called the E-G sequence). In seeking the sequencing effect, we focused only on the difficulties caused by the change in representation of problems. The average numbers of difficulties caused by representational change that each student encountered in each sequence in interviews 2 and 4 are given in Tables 1 and 2, respectively.

The first trend that we ought to point out is that the students always had more difficulties with the first problem than the second. This indicates that students learn skills in the first problem that they then apply to the second problem. Not a surprising result; however, it is important to note that improvement in skills occurs in training.

A sequencing effect could be observed in each interview when we compared the average number of difficulties each student encountered in the G-E sequence with the E-G sequence. In interview 2, each student had an average of 5.25 difficulties in the G-E sequence while having only 1.5 difficulties in the E-G sequence. This suggested that students had fewer difficulties if they attempted the equation problem first and the graphical problem later. However, the data from interview 4 showed the opposite effect. Each student had an average of 0.75 difficulties in the G-E sequence while having 2.84 difficulties in the E-G sequence. This apparent contradiction could be explained as follows. In the graphical problem in interview 2, students could find potential energy stored in the spring by either finding the spring constant and spring compression from the graph to plug into '$\frac{1}{2}kx^2$' or finding the area under the graph. When given the graphical problem first, most students attempted the first method in which they encountered some difficulties reading off information from the graph (R-D-GRA-INFO). In contrast, when given the equation problem first, students only had a few difficulties finding work from the force equation because it was observed that students had a tendency to integrate whatever variable was changing. This strategy appeared to have been productive in this problem even though students did not really understand the physical meaning of integration. Then when these students moved onto the graphical problem, they were able to recognize the method of finding the area under the graph.

Student: *"This problem also has friction but it tells you the friction by a graph of the equation."*

Interviewer: *"Okay, so how would you find work in this problem?"*

Student: *"It would be the area."*

Interviewer: *"Which area?"*

Student: *"The area under the curve."*

This decreased the number of difficulties students encountered in the graphical problem, which in turn decreased the average number of difficulties.

In interview 4, the situation was reversed when the graphical problem was more straightforward than the equation problem. The graph provided in this interview was not linear, so none of the students attempted to find “coefficient of friction” as they did with the graph in interview 2. Instead they went on to find the area under that graph and had no difficulty with this task. After calculating the area (in unit of Newton times degrees), students had some difficulties with converting units to get to the right unit of work, which is Newton times meter. The equation of rolling friction force as a function angle given in interview 4 was not easy to handle for those students who did not really understand the concept of function and the physical meaning of integration. Some students did not know what to do with such a function. Some asked the interviewer whether it meant “F is a function of θ” or “F times θ”. Another difficulty came from finding work done by friction, for which purpose an integral of F(θ).dθ was not enough because the correct integral for work should be that of ‘F(θ)ds’, in which ‘ds = R.dθ’.

Student: *“I’m not sure what to do with this function … Is this ‘F times θ’ or ‘F is a function of θ’?”*

Interviewer: *“F is a function of θ.”*

Student: *“So I should take derivative or integral of F.”*

Interviewer: *“Okay, so derivative or integral?”*

Student: *“Derivative … I guess.”*

Interviewer: *“You need to integrate force to find the total work done by that force. So which integral should you take?”*

Student: *“Integral of F.”*

Interviewer: *“With what variable?”*

Student: *“Theta”*. [Wrote down $\int F(\theta)d\theta$]

All students had difficulties making use of the function and could only set up the right integral after several hints from interviewer. For students following the G-E sequence in interview 4, they had no major difficulty finding the area under the graph, but had some difficulties converting the unit afterward. These difficulties were not due to the representational change so they were discounted in our analysis of the sequencing effect, although those difficulties helped students with the equation problem that came later in which they could take an integral of F(θ).dθ to get the area under the graph of F(θ) versus θ and then convert units to get work. For students following the E-G sequence, the difficulties with units were actually included in the equation because after knowing that an integral of F(θ).dθ was not the correct one for work, students tried to set up the right integral of F(θ).ds which then led them directly to the correct value of work. This task increased the average number of difficulties per student with the equations representing functional relationships.

Emergent Themes in Students' Performance

Applying Case-reuse Strategies.

We observed in our interviews that students tried to mimic previous problems whenever possible. This behavior was expected and could also be productive when used appropriately (Jonassen, 2006). However, in most cases in our interviews, reusing the original problem was not a productive strategy for students while they were trying to solve the graphical and equation problems.

The original problem of interview 2 gave students a linear spring with the spring constant k provided. Students found the spring potential energy using the formula $U = (1/2)kx^2$. Then in the graphical problem, students were provided with a linear graph of spring force *F* (in Newtons) versus the spring's compression *x* (in meters). Students did not find the potential energy stored in the spring by calculating the area under the graph of *F* vs. *x* as we would have preferred. Instead, they tried to find spring constant *k* so that they could reapply $U = (1/2)kx^2$. In this situation, the case-reuse strategy (Jonassen, 2006) helped because the spring constant could be calculated as the slope of the graph of F vs. x, due to the relation $F = kx$. However, the case-reuse strategy turned out to be inappropriate as students moved to the equation problem in which they were given a non-linear spring whose force depends on the compression as per the equation $F = 1000x + 3000x^2$. Most students tried to find the spring constant using the formula $k = F / x$. As students obtained the result $k = 1000 + 3000x$, they were asked the question "*Is your 'spring constant' a constant*?" which helped them recognize that they could not find the spring potential energy simply by applying $U = (1/2)kx^2$ as they had done in the previous problems. Most students were unable to proceed beyond this point. It appeared that students lacked the necessary knowledge and skills to realize that the non-standard equation of force as a function of displacement was itself a cue that Hooke's Law was not applicable and that they could not simply find the force constant.

They were prompted to refer back to the graphical problem where they had been guided to find the spring potential energy without knowing the spring constant by finding the area under the graph. Then the question "*The effect of which operator on a function is equivalent to finding the area under the graph of that function?*" seemed to trigger students to think of performing an integral of the function to find the spring potential energy.

> [Student found k = F/x = 1000 + 3000x, then plugged in ½ kx^2 to find spring potential energy. Then he had the equation: ½ mv^2 = ½ (1000 + 3000x)$(.2)^2$ – mgh.]
>
> Interviewer: *Now you have an equation with v and x, how do you solve it?*
>
> Student: *[pause] So we need another way.*

Interviewer: *Can you think of anything we used in the previous problem that would help?*

Student: *No.*

Interviewer: *Well, in the graphical problem we talked about finding the potential energy of a spring without knowing k and x explicitly.*

Student: *The area.*

Interviewer: *What operator is equivalent to finding the area?*

Student: *Integral.*

Interviewer: *Can you set up an integral in this problem?*

Student*: I think I can.*

Extracting Information from a Graph.

It was also observed in our interviews that most students needed help extracting the information they needed from the graph. This information included the coordinates of a point, the area under the graph, the value of a physical quantity that each division on the graph represents, etc. Some students were able to read off explicit information from the graph such as the quantities being plotted or the coordinates of a point, but then did not know how to go further with that information.

The idea of calculating the slope of the graph seemed to appear automatically in the students' minds whenever they were provided a graph, even when the slope had nothing to do with the quantity they wanted to solve for. This helped students find the spring constant *k* in the graphical problem of interview 2 as mentioned above, but it caused students difficulty as they moved on to the graphical problem of interview 4 in which there was a non-linear graph and the slope was changing continuously along the curve. By interview 4, however, students were able to recognize that the area under the curve was the work done by friction since they had seen those kinds of problems in interviews 2 and 3. Because the graph in problem 4 was not a straight line, we expected students to suggest a method to obtain a good estimation of the area under the graph. Some students divided the area into several smaller pieces by vertical lines and then divided each piece into a triangle and rectangle to calculate the areas. Others counted the number of squares under the graph. About two thirds of the students who counted the squares reported the number of squares under the graph as the area. They forgot to take into account the physical quantity each square represented, which in this problem was 2.5 Newton times degrees.

Physical Meaning of Differentiation and Integration.

Throughout our interviews, students appeared to have adequate mathematics knowledge needed to solve our interview problems, but were unable to apply that knowledge in the contexts of physics problems. An example was the physical meaning of differentiation and integration. It was observed that students had significant difficulty setting up the correct integral to calculate the desired physical quantity, but then computed that integral very easily. When students were given a function $f(x)$ and realized that they needed to integrate that function, the integral students set up was $\int f(x)dx$, regardless of what $f(x)$ represented and what they were looking for. So, hints on meanings of differentiation and integration did not make much sense to students. Rather, hints on some basic things like units of quantities seemed to help students better. Analyzing the equation problem in interview 4 (Figure 2) provides a good example for this situation.

In the equation problem of interview 4, students were given an equation of rolling friction force as a function of angle. To be able to apply conservation of energy, students must calculate the work done by this friction, which could only be done by computing an integral of the force function. Recalling that integrating means chopping and adding up little pieces, one recognizes that he must divide the curve into small segments, find the work done by friction on the relevant segment, and add up the work over all the pieces. Mathematically, the work done by friction on a small segment of the curve is $F\left(\theta\right).ds$ in which $ds = Rd\theta$ is a small segment of length along the curve. Then the integral for work is $\int_0^{\pi/2} F(\theta)Rd\theta$.

As discussed above, most of them attempted to compute the integral $\int_0^{\pi/2} F(\theta)d\theta$ (similar to $\int f\left(x\right)dx$). None of the students expressed any doubt on what F(θ) and $\int_0^{\pi/2} F(\theta)d\theta$ represented. Physically, the integral $\int_0^{\pi/2} F(\theta)d\theta$ meant adding up all of the infinitesimal segments of a quantity defined as force times angle, which had no physical meaning. In this case, with the first few students we tried to guide them to convert $d\theta$ to ds by the relation $ds = Rd\theta$ so that they had the correct integral. Students followed our hints to do so but said that they did not understand why they had to do that step. We recognized that in this problem, the factor R (radius of the curve) didn't need to come into the integral at the beginning but instead could be multiplied with the result of $\int_0^{\pi/2} F(\theta)d\theta$ at the end of the process. So for later students, we let them compute the integral $\int_0^{\pi/2} F(\theta)d\theta$ they had set up by themselves until they obtained a number. Students were then asked what the unit of that number was. Most of them were able to say that the unit of their result was Newton times Radians. Through appropriate questioning we enabled students to recognize that Newton times Radians was not the unit of work and guided them to convert that unit into Newton times meters. During this process students recognized the missing factor R (radius of the track) in their calculations.

Summary

We conducted individual teaching/learning interviews with students in a first-semester calculus-based physics course to investigate the kinds of difficulties students had when solving problems in numerical, graphical, and equation representations and the kinds of hints that might help them overcome those difficulties. We found that students encountered many kinds of difficulties when solving our interview problems. Beside the difficulties with applying physics knowledge to a problem (not knowing what physical principle to use to solve the problem, not remembering the formula of a physical quantity, misunderstanding the terms in a formula, etc.), there were also difficulties in applying mathematical ideas and processes into the context of a physics problem (interpreting the physical meaning of the slope of a graph, finding work done by a changing force by integrating the force function or estimating the area under the graph of force versus distance). With proper hints given by the interviewer, all of the students were able to solve the problems correctly. Explicit hints (statements asking students to refer to some information or knowledge or to perform a specific calculation) seemed to help students get started on each problem, while more implicit hints (questions asking students to reflect on what physics knowledge and mathematical process was applicable in each physical situation) seemed to trigger students' thinking of the physical meaning of mathematical processes. It might not be surprising that students used the case-reuse strategy whenever possible to solve the problem, although that strategy was not always productive in our interviews. It would be interesting to find out that the sequence of problems in multiple representations given to students seems to have a certain effect on their performance in our interviews, although it still needs to be investigated in more detail before a conclusion can be reached.

Implication

This study provided a closer look at students' difficulties when solving physics problems in multiple representations. It constitutes a research-based database on which an online system can be built to better address students' needs for assistance when solving physics problems in numerical, graphical, and equation representations.

This study also contributes to the body of knowledge in the area of use of multiple representations in solving problems in introductory physics. It informs us of the barriers that students encounter as they progress to representational competence, which is an important skill that future scientists and engineers should have. It also suggests the kinds of scaffolding that might be helpful for students when solving problems in different representations.

This study targets an important issue in science education: preparing future scientists and engineers with the competence to solve problems in a variety of representations. It provides ideas on the kinds of scaffolding instructors and curriculum developers might include in their instruction or curricula to facilitate students' representational competence.

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Table 1

Majors of Students Participating in our Interviews

Major	**Number of students**
Mechanical Engineering	6
Electrical Engineering	5
Civil Engineering	2
Chemical Engineering	3
Architectural Engineering	1
Environmental Engineering	1
Chemistry	1
Open Option	1

Table 2

Average Number of Difficulties per Student Due to Representational Change in Interview 2

	G-E sequence	E-G sequence
Graphical problem	3.50	0.17
Equation problem	1.75	1.33

Table 3

Average Number of Difficulties per Student Due to Representational Change in Interview 4

	G-E sequence	E-G sequence
Graphical problem	0.50	0.17
Equation problem	0.25	2.67

Appendix

Interview 1

Problem 1

The position of an object moving along an *x* axis is given by $x = 3t^3 - 2t + 4$, where *x* is in meters and *t* in seconds.

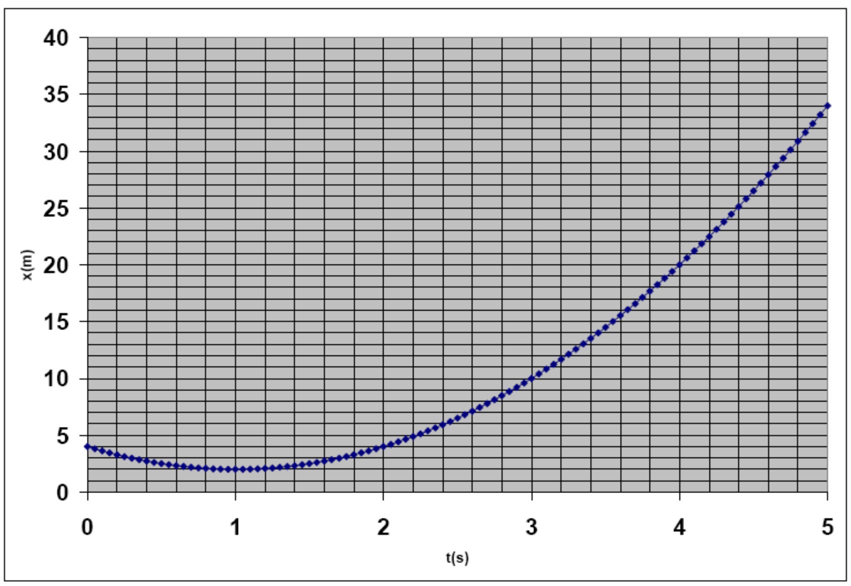
a) Find at least one time when the velocity is zero.

b) What is the average acceleration between 0 and 3 seconds?

c) What is the acceleration at $t = 3$ seconds?

Problem 2

The position of an object moving along an *x* axis versus time is given by the graph below, where *x* is in meters and *t* in seconds.



a) Find at least one time when the velocity is zero.

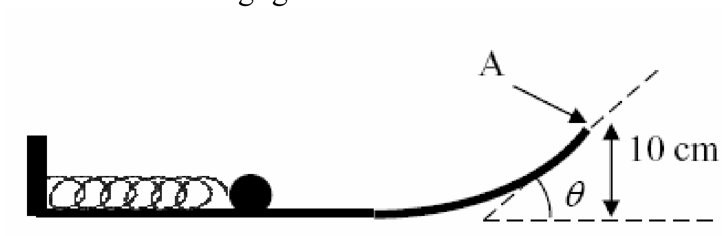
b) What is the average acceleration between 0 and 5 seconds?

c) What is the acceleration at $t = 3$ seconds?

Interview 2

Problem 1

A spring of spring constant 3.0 kN/m is compressed a distance of 1.5 cm and a small ball is placed in front of it. The spring is then released and the small ball, mass 0.1 kg, is fired along the slope and launched into the air at point A which is 10 cm above the spring. The angle θ of velocity at launch is 30°. Friction is negligible.

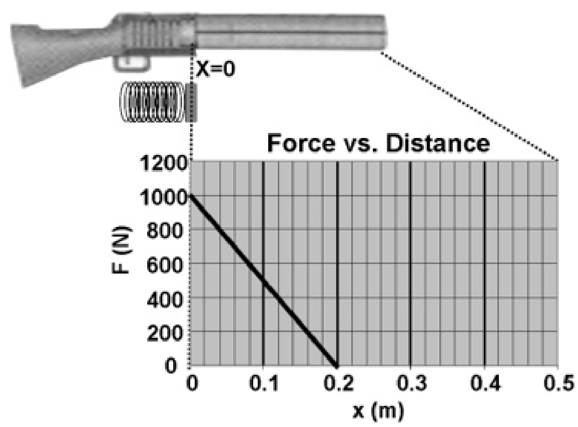


What is the speed of the ball at the launch point (point A)?

Problem 2

A 0.1 kg bullet is loaded into a gun (muzzle length 0.5 m) compressing a spring as shown. The gun is then tilted at an angle of 30° and fired.

The only information you are given about the gun is that the barrel of the gun is frictionless and when the gun is held horizontal, the net force F (N) exerted on a bullet by the spring as it leaves the fully compressed position varies as a function of its position x (m) in the barrel as shown in the graph below.

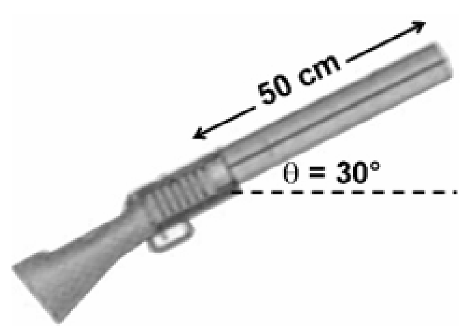


What is the muzzle velocity of the bullet as it leaves the gun, when the gun is fired at the 30° angle as shown above?

Problem 3

A 0.1 kg bullet is loaded into a gun (muzzle length 0.5 m) compressing a spring to a maximum of 0.2 m as shown. The gun is then tilted at an angle of 30° and fired.

The only information you are given about the gun is that the barrel of the gun is frictionless and that the gun contains a non-linear spring such that when the held horizontal, the net force, F (N) exerted on a bullet by the spring as it leaves the fully compressed position varies as a function of the spring compression, x (m) as given by: $F = 1000x + 3000x^2$

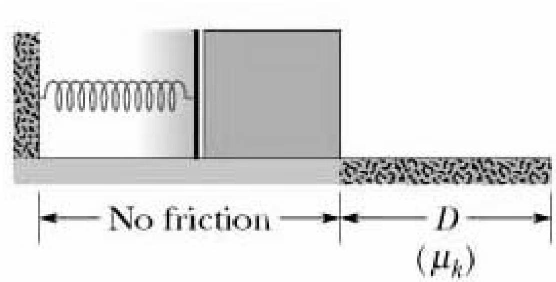


What is the muzzle velocity of the bullet as it leaves the gun, when the gun is fired at the 30° angle as shown above?

Interview 3

Problem 1

A 3.5 kg block is accelerated from rest by a spring, spring constant 632 N/m that was compressed by an amount x. After the block leaves the spring it travels over a horizontal floor with a coefficient of kinetic friction $\mu_k = 0.25$. The frictional force stops the block in distance D = 7.8 m.

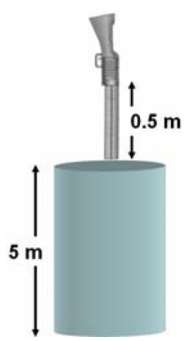


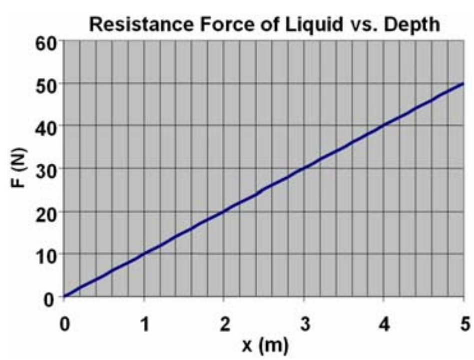
What was the spring compression *x*?

Problem 2

A 0.1 kg bullet is loaded into a gun compressing a spring of spring constant k = 6000 N/m. The gun is tilted vertically downward and the bullet is fired into a drum 5.0 m deep, filled with a liquid.

The barrel of the gun is frictionless. The resistance force provided by the liquid changes with depth as shown in the graph below. The bullet comes to rest at the bottom of the drum

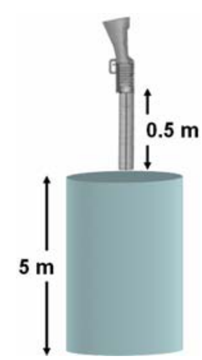




What is the spring compression *x*?

Problem 3

A 0.1 kg bullet is loaded into a gun compressing a spring of spring constant k = 6000 N/m. The gun is tilted vertically downward and the bullet is fired into a drum 5.0 m deep, filled with a liquid.



The barrel of the gun is frictionless. The frictional force *F*(N) provided by the liquid changes with depth *x*(m) as per the following function.

$$F = 10x + 0.6x^2$$

The bullet comes to rest at the bottom of the drum. What is the spring compression *x*?

Interview 4

Problem 1

A hoop radius r = 1 cm and mass m = 2 kg is rolling at an initial speed v_i of 10 m/s along a track as shown. It hits a curved section (radius R = 2.0 m) and is launched vertically at point A.
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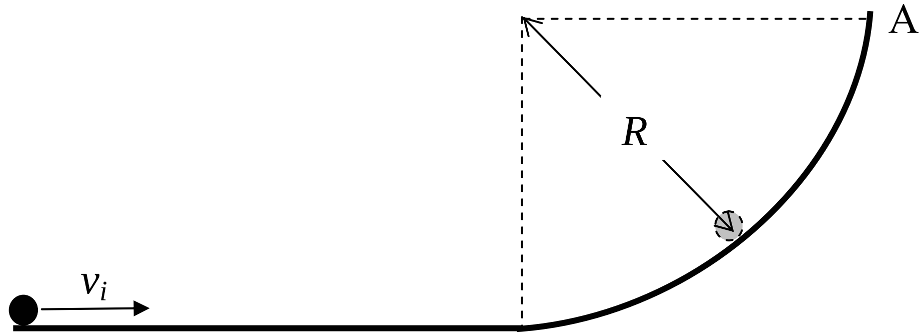
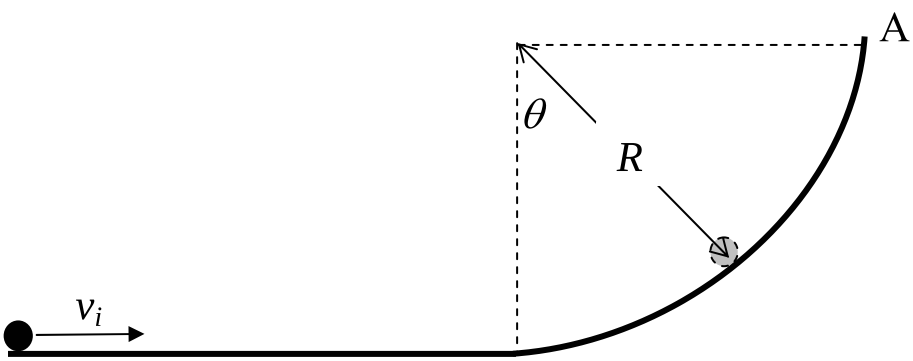


What is the launch speed of the hoop as it leaves the slope at point A?

Problem 2

A sphere radius r = 1 cm and mass m = 2 kg is rolling at an initial speed v_i of 5 m/s along a track as shown. It hits a curved section (radius R = 1.0 m) and is launched vertically at point A. The rolling friction on the straight section is negligible.



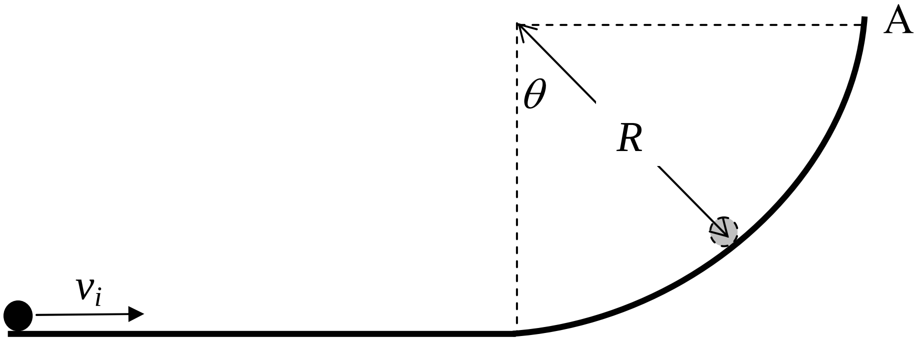
The magnitude of the rolling friction force F_{roll} (N) acting on the sphere varies as angle θ (radians) as per the following function

$$F_{roll}(\theta) = -0.7\theta^2 - 1.2\theta + 4.5$$

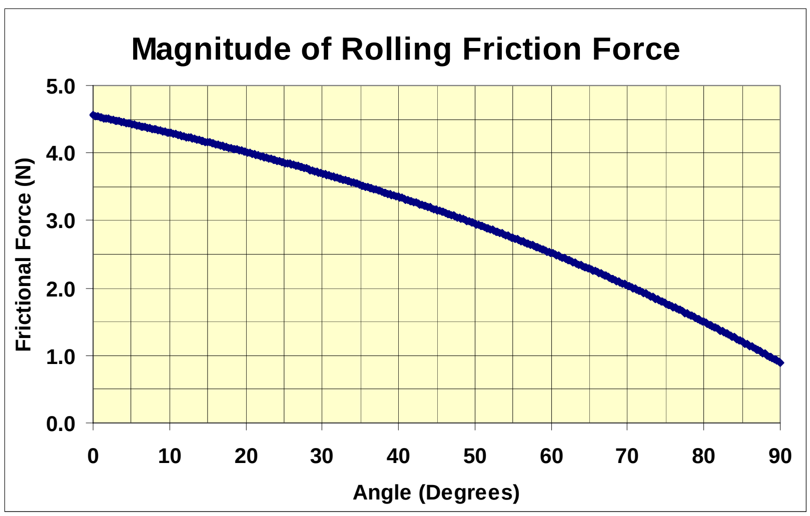
What is the launch speed of the hoop as it leaves the curve at point A?

Problem 3

A sphere radius r = 1 cm and mass m = 2 kg is rolling at an initial speed v_i of 5 m/s along a track as shown. It hits a curved section (radius R = 1.0 m) and is launched vertically at point A. The rolling friction on the straight section is negligible.



The magnitude of the rolling friction force acting on the sphere varies as angle θ as per the graph shown below



What is the launch speed of the hoop as it leaves the curve at point A?