Transfer of Learning in Problem Solving in the Context of Mathematics and Physics

N. Sanjay Rebello, Lili Cui, Andrew G. Bennett, Dean A. Zollman & Darryl J. Ozimek

Introduction

Transfer of learning, which has sometimes been considered to be the ultimate goal of education, (McKeough, Lupart, & Marini, 1995) is often described as the ability to apply what one has learned in one situation to a different situation. (Reed, 1993; Singley & Anderson, 1989) Problem solving involves transfer in several ways. Problem solving in semi-structured and unstructured domains often involves the transfer of knowledge and skills from a structured (classroom) domain to the semi-structured or unstructured domain. Furthermore, even within a structured domain, a standard heuristic for developing a strategy is to ask if a similar problem has been encountered in the past and to see if a similar technique will work for the new problem. (Pólya, 1957) Until recently, several researchers who have studied transfer of learning in the context of sequestered problem solving have often found that transfer is rare. (Duncker, 1945; Gick, 1980; Reed, Ernst, & Banerji, 1974) Most students are unable to recognize similarities between the learning context and the transfer context, whether within a structured domain or moving from a structured to a semi-structured or unstructured domain, and are therefore unable to successfully solve problems in the latter context, even though they may have been trained to do so in the original learning context. Researchers have often explained the lack of such transfer in terms of students' inability to construct a coherent schema in the learning domain to begin with. (Reed, 1993) Lack of a robust schema in the initial domain impedes students' abilities to apply their knowledge in new domains.

Contemporary Views of Transfer

Recently researchers (Bransford & Schwartz, 1999; Greeno, Moore, & Smith, 1993; Lobato, 1996) have begun to expand their notions of transfer and how to assess them. Rather than examine students' abilities to successfully solve a problem in the new domain, researchers have been examining students' abilities to *learn* how to solve problems in the new domain. (Bransford & Schwartz, 1999)

In addition to the cognitive aspects affecting transfer, researchers have also been paying attention to the mediating factors such as students' epistemologies and expectations. (diSessa, 1993; Hammer & Elby, 2002) Also, rather than focus on robust schemas to describe transfer, researchers have been focusing on activation of pieces of knowledge (diSessa, 1988) or cognitive resources (Hammer, 2000) In the new domain and the dynamic construction of similarities between the learning and transfer context. (Lobato, 1996) Our research (Rebello *et al.*, 2005) has combined these contemporary perspectives of transfer to construct a model that explains the dynamic transfer of learning which occurs as we conduct a 'think-aloud' interview involving a problem solving task. The model enables us to gain insights into students' thinking processes as they solve problems in unfamiliar domains.

Our model of transfer, described in detail in a recent publication, (Rebello *et al*., 2005) encapsulates several of the contemporary views of transfer described above. As per this model, transfer is the dynamic creation of associations between a learner's prior knowledge and information that is 'read-out' by the learner from a new situation. Read-out of information as well as activation of prior knowledge is controlled by a learner's epistemic mode, motivation and other mediating factors. This model of transfer does not make distinctions between productive and unproductive associations that a learner might make in a given situation, rather it examines all possible associations that a learner might make in a given situation. Therefore, the model describes the dynamics of the process of knowledge construction in a new situation. In this chapter we apply our model of transfer to the process of problem solving. We discuss two qualitatively different types of transfer processes that we believe learners use when solving problems.

Research Contexts

This chapter focuses specifically on transfer of learning from mathematics to physics contexts. Although all of the problem contexts that we examine are associated with classroom learning, the physics contexts are more closely connected with real-world unstructured problems than typical mathematics problems. Therefore, by examining students' transfer of learning from mathematics to physics we can gain insights into the types of barriers students might face if they attempt to transfer their learning to even more unstructured domains such as authentic real-world problems. In other words, if students are unable to transfer what they have learned from mathematics to physics they are extremely unlikely to transfer successfully what they have learned to authentic real-world situations.

The studies discussed later in this chapter focus on transfer from mathematics to physics courses. The first investigates transfer of learning from a calculus course to a calculus-based physics course taken primarily by engineering and physics majors. The second study focuses on transfer of learning from a trigonometry course to an algebra and trigonometry-based physics course taken primarily by life science majors. We use a combination of qualitative and quantitative methods to examine transfer and problem solving. The qualitative studies utilize data collected through individual clinical interviews as well as teaching interviews. (Engelhardt, Corpuz, Ozimek, & Rebello, 2003; Steffe & Thompson, 2000) While the former aim to examine students' states of knowledge and the ideas that they spontaneously transfer to a new problem situation, the latter shed light on how students assemble their knowledge elements to construct problem solutions and new knowledge in previously unfamiliar situations. In addition to clinical and teaching interviews, we also examine students' transfer of learning through quantitative analysis of correlations between online homework and exam scores in mathematics and physics and engineering classes. (Ozimek, 2004) A significant correlation between scores on two temporally separated components of evaluation might indicate an underlying factor that is common to both contexts and therefore might point to hidden associations that students have activated in the new context based on their familiarity with the previous context.

Chapter Overview

We utilize transfer of learning as a lens through which we examine students' problem solving in previously unfamiliar domains. In the first section we begin with a review of the multiple perspectives that have been used to study transfer as well as the factors that affect it. In the second section, we identify the particular perspectives of transfer that shape our research and describe a theoretical framework that serves as a lens with which to analyze our research results. In the last two sections we examine students' problem solving and transfer from a relatively more structured domain -- an undergraduate mathematics course to a somewhat less structured domain -- an undergraduate physics course. Through a combination of quantitative and qualitative methods, we gain insights into the factors that mediate students' transfer of learning and problem solving in physics. We interpret these insights from the perspective of our theoretical framework on transfer.

Multiple Perspectives on Transfer

Traditionally, transfer of learning is often (Reed, 1993; Singley & Anderson, 1989) defined as applying what one has learned in one situation to another situation. Due to the lack of evidence of transfer in many studies based on traditional models, recent views of transfer have shifted to look at transfer from other perspectives.

Traditional models (Adams *et al.*, 1988; Bassok, 1990; Brown & Kane, 1988; Chen & Daehler, 1989; Nisbett, Fong, Lehmann, & Cheng, 1987; Novick, 1988; Perfetto, 1983; Reed, 1993; Singley & Anderson, 1989; Throndike & Woodworth, 1901; Wertheimer, 1959) are based on a researcher's pre-defined concept that they hope students will transfer. These models also view transfer as a static passive process. The traditional models of transfer have tended to focus on the cognitive aspects of transfer. Thorndike's theory of identical elements asserts that training in one kind of activity transfers to another only if the activities share common elements; which are generally taken to mean identical at the level of the surface features of the stimulus environment. (Throndike & Woodworth, 1901) According to Judd's theory of deep structure transfer it depends upon the extent to which the learner notices underlying shared causal principles between two problems. (Judd, 1908) More recently, as per the information processing perspective, transfer is mediated by abstract, symbolic mental representations. (Singley & Anderson, 1989) The learner constructs an abstract mental representation or schema through experiences in the learning situation and deploys the schemas in the transfer situation.

Contemporary models of transfer have gone beyond focusing solely on the cognitive aspects of transfer. Rather they have included several other mediating factors that affect transfer. The socio-cultural perspective asserts that the social and cultural environment affects transfer through language, cultural tools and more knowledgeable individuals such as parents, teachers or other domain experts. Transfer in terms of affordances and constraints of activity focus on the extent to which participating in an activity while being attuned to the affordances and constraints in one situation influences the learner's ability to participate in a different situation. (Greeno *et al.*, 1993) The actor-oriented perspective conceives transfer as the personal construction of similarities between activities where the 'actors,' i.e. learners, see situations as being similar. (Lobato, 1996) Preparation for future learning focuses on whether students can *learn* to solve problems in transfer situations in a similar way in which they initially learned the content, i.e. using available resources. (Bransford & Schwartz, 1999) Contemporary models of transfer (Bransford & Schwartz, 1999; Greeno *et al.*, 1993; Lobato, 2003) account for aspects the traditional models neglect. They take into account the socio-cultural factors that mediate transfer and view transfer from the students' points of view rather than the researcher's point of view. A common feature of all of these perspectives is that they consider transfer as an active dynamic process. In the next section we briefly describe a model of transfer that is consistent with these contemporary perspectives. Based on the model, we construct a theoretical framework that distinguishes between different kinds of transfer processes relevant to problem solving.

Theoretical Framework

Two Kinds of Associations

Our model of transfer, which is based on a framework presented by Redish (Redish, 2004) and earlier cognitive psychologists, views transfer as the dynamic creation of associations by the learner in a new problem situation. This model provides a useful lens in examining transfer from the contemporary perspectives discussed above. We believe that there are two kinds of associations that a learner can create in a problem solving scenario.

The first kind of association involves assigning information read out from a problem to an element of the learner's prior knowledge. An example is reading out a numerical value from the problem statement and assigning it to a particular physical quantity. For instance, if a problem states that a car is moving at 20 meters/second, the learner recognizes that the 20 meters/second is the car's 'velocity' and more specifically that '$v = 20m/s$' must be plugged into a particular equation. The equation in this case is a part of the learner's internal schema to solve the problem. These kinds of associations – between new information gleaned from the problem and elements of the learner's internal knowledge structure are usually firmly established in the learner's mind and easily articulated by the learner. A second kind of association occurs between a knowledge element read-out from the problem with an element of the learner's internal knowledge structure, which in turn is based on their prior knowledge. This association is usually more abstract and tenuous and often the learner may not be able to clearly articulate it. For instance a student who is shown an animation of a moving car, without being even told that velocity has anything to do with problem, begins to think about the car's velocity as an important feature of the problem. This learner is making an implicit association between two ideas – motion (shown in the problem animation) and velocity (knowledge of which is deemed necessary to describe the motion).

Two Kinds of Transfer

We believe that it is important to distinguish between these two flavors of associations that a learner might make in a problem scenario because they are tied to two different kinds of transfer processes. In the first kind of transfer -- 'horizontal' transfer -- the learner reads-out explicitly provided information from a problem scenario that activates a pre-created knowledge structure[1] that is aligned with the new information read out from the problem. This alignment between the provided information and the learner's knowledge structure determines whether the learner can solve the problem. If such alignment or assignment does not naturally occur, i.e. if the external problem representation does not match the learner's knowledge structure or internal problem representation, the learner is unable to solve the problem. A typical example of horizontal transfer occurs when learners solve 'plug-and-chug' problems at the end of chapters in some science and mathematics textbooks. The learner reads the problem statement, which explicitly provides information in terms of the required variables, e.g. the initial velocity, acceleration and time of a moving vehicle and clearly states the goal of the problem such as finding the displacement of the vehicle. Upon reading out this information from the problem, the learner activates a particular equation of motion from their memory. In this case this equation is the learner's internal schema or mental model for solving this problem. The learner plugs the variables from the problem into this equation to solve the problem. Neither does the learner need to consider the underlying assumptions of the equation that determines the situation in which the equation is applicable nor does the learner have to choose between several different equations to decide which is used in this problem situation. Several end-of-chapter problems in textbooks fall under this category. The problem statement often explicitly provides all of the required information and no more. The equation or representation that is applicable to the problem scenario can often be found in the text by matching the information provided in the problem to a

[1] The term 'internal knowledge structure' refers to a pre-created set of tightly associated knowledge elements. Other terminology that is often used in literature includes 'schema', 'internal representation' 'mental model' or

limited set of equations and finding one that matches – often a pattern matching task. The learner is never called upon to critically examine the situation or the assumptions underlying the model (i.e. equation) that they use to solve it.

In the second kind of transfer -- ‘vertical’ transfer -- a learner recognizes features of the situation that intuitively activate elements of her/his prior knowledge. In this type of transfer the learner typically does not have a preconceived knowledge structure that aligns with the problem information. Rather, the learner constructs a mental model *in situ* through successive constructions and deconstructions of associations between knowledge elements. For instance, rather than being told the initial velocity and acceleration of the vehicle the learner is shown a video clip or animation of the vehicle and asked to find out how much farther the vehicle may have traveled after going off the edge of the video clip. Nowhere is the learner told the initial velocity or acceleration or even that these variables are relevant to the situation. In this case, the leaner first must recognize that the vehicle was accelerating and may even confront the assumption that this acceleration may not be uniform. Nowhere is any hint provided about the equation that must be used or even that an equation may be applicable. So the learner cannot activate a clearly identifiable preconceived knowledge structure or internal representation that neatly aligns with the situation. At the very least, the learner must choose between competing internal representations or construct a new one for this situation. Choosing the most productive internal representation from several representations depending upon the problem situation is a key feature of ‘vertical’ transfer. Few, if any problems in most science or mathematics textbooks require vertical transfer. Most real-world problems require ‘vertical’ transfer. Often the

---

‘coordination class.’

problems are too complex to solve without neglecting some confounding features or variables. Solving real-world problems requires learners to decide which variables can be neglected and also deciding what schema or model is applicable under those assumptions or creating one specially for the situation. Real-world problem solving also requires students to know the limitations of the model that they have decided to use and under what hypothetical conditions the model would be no longer applicable.

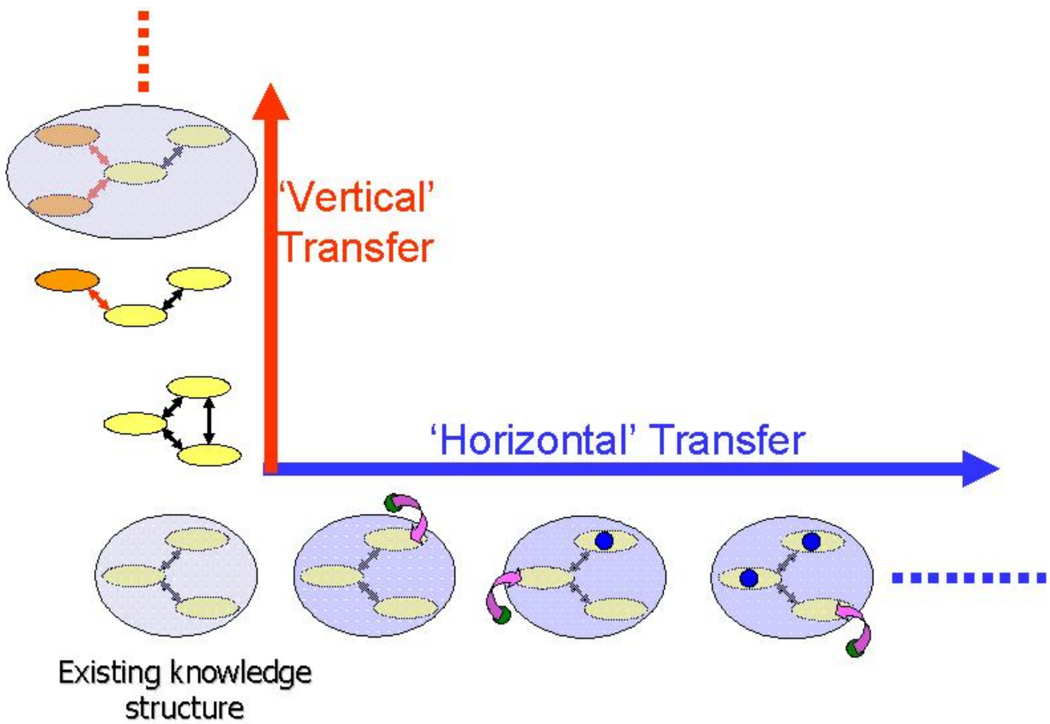

Figure 1: 'Horizontal' transfer involves activation and mapping of new information onto an existing knowledge structure. 'Vertical' transfer involves creating a new knowledge structure to make sense of new information.

Figure 1 shows the difference between 'horizontal' and 'vertical' transfer. We find that that the graphical metaphor with a horizontal and vertical axis to represent the two kinds of transfer is a useful pictorial representation to highlight the distinctiveness of the two kinds of transfer processes. It also is useful in representing the notion that a given process can have components of both ‘horizontal’ and ‘vertical’ transfer and that these two processes are not mutually exclusive in any way.

Similar Views from Others

The notions of ‘horizontal’ and ‘vertical’ transfer described above are not new. Indeed there is a vast body of literature on knowledge and conceptual change that expresses ideas along these lines. Several decades ago Piaget (Piaget, 1964) proposed two mechanisms of conceptual change – ‘assimilation’ in which new information was incorporated into a learner’s internal knowledge structure without modification and ‘accommodation’ in which new information led to a modification of the learner’s internal knowledge structure. Although Piaget’s ideas focused on conceptual change and not on transfer per se, we believe that the mechanisms of assimilation and accommodation align closely with ‘horizontal’ and ‘vertical’ transfer respectively. Similar ideas are also expressed by Broudy (Broudy, 1977) who identifies at least two kinds of knowing – “knowing what” or ‘applicative’ knowing versus “knowing with” or ‘interpretive’ knowing. The former includes clearly articulated procedures or schema that a learner uses in a given situation. The latter, which is much more subtle and intangible, refers to a sense of intuition or ‘gut instinct’ that a learner brings to bear as he/she makes sense of a new situation and frames the problem. We believe that Broudy’s notions of applicative and interpretive knowing align closely with our ideas of ‘horizontal’ and ‘vertical’ transfer respectively. Much more recently, diSessa and Wagner (diSessa & Wagner, 2005) have discussed transfer in light of their coordination class theory of conceptual change. DiSessa had previously (diSessa, 1998) described the theory of a ‘coordination class’ -- a class or concept that allows the learner to read out and process information from the real world. In their more recent article, diSessa and Wagner (diSessa & Wagner, 2005) apply their coordination class theory to elucidate their perspective on transfer of learning. They distinguish between what they call ‘Class A’ and ‘Class C’ transfer. ‘Class A’ transfer, which we believe is analogous to ‘horizontal’ transfer, occurs when a learner applies “well prepared” knowledge such as a coordination class to a new situation. ‘Class C’ transfer, which we believe is analogous to ‘vertical’ transfer, occurs when “relatively unprepared” learners use prior knowledge to construct new knowledge. In a sense, ‘Class C’ transfer is indistinguishable from learning. There is at least one point of caution in making the comparisons between our classification and the one used by diSessa and Wagner. diSessa and Wagner’s assertion that ‘Class C’ transfer happens all the time while ‘Class A’ transfer is relatively rare might appear to contradict our earlier comparisons between ‘horizontal’ and ‘vertical’ transfer. On closer examination however we believe that there is in fact no contradiction. Although learners continuously process new information and construct knowledge, (‘Class C’ transfer) it is relatively difficult for learners to construct knowledge that is useful, “well prepared” or easily applicable later. (‘Class A’ transfer) So, the reason ‘Class A’ transfer is rare is because the learner does not possess “well prepared” knowledge. ‘Class C’ transfer must precede ‘Class A’ transfer and while the former may occur all the time, it does not necessarily yield “well prepared” knowledge that is required for ‘Class A’ transfer.

Our notion of ‘horizontal’ and ‘vertical’ transfer is also similar to classifications by other transfer researchers. Salomon and Perkins (Salomon & Perkins, 1989) distinguish between ‘low road’ and ‘high road’ transfer. ‘Low road’ or more typically ‘near’ transfer occurs when the scenario in which original learning had occurred is similar to the new problem scenario so that the learner can successfully apply preconceived problem-solving processes. ‘High road’ or more typically ‘far’ transfer is much more challenging in that it requires the learner to abstract the new situation and engage in reflection and metacognition to help construct a way to solve the problem. The distinction between ‘low road’ and ‘high road’ transfer is akin to the distinction between ‘horizontal’ and ‘vertical’ transfer respectively. A similar distinction is made by Bransford and Schwartz (Bransford & Schwartz, 1999) when they compare two measures of transfer – ‘sequestered problem solving (SPS)’ and ‘preparation for future learning (PFL).’ While the former measure focuses on whether students can directly apply their learning to a new situation ‘cold,’ i.e. without any scaffolding or support, the latter measure focuses on whether their learning has prepared them to learn in the future. To measure transfer as per the PFL perspective we must observe whether a learner can bring to bear their earlier experiences and learn to construct a solution to their new problem. Bransford and Schwartz point out that most traditional transfer measures focus on SPS rather than PFL and consequently fail to find evidence of transfer. More recently, Schwartz, Bransford and Sears (Schwartz, Bransford, & Sears, 2005) have contrasted the notions of ‘efficiency’ and ‘innovation’ in transfer. ‘Efficiency’ refers to a learner’s ability to rapidly recall and apply their knowledge in a new situation while ‘innovation’ is their ability to restructure their thinking or reorganize the problem scenario so that it becomes more tractable than before. We believe that developing ‘efficiency’ in problem solving is analogous to engaging in ‘horizontal’ transfer while ‘innovation’ is analogous to ‘vertical’ transfer.

Finally, the notion of horizontal and vertical transfer has often been used by researchers in problem solving who distinguish between ‘well-structured’ and ‘ill-structured’ problems. (Jonassen, 2000, 2003) Well-structured problems have clearly defined information and goals. Therefore, they are akin to problems that require mainly, if not only, ‘horizontal’ transfer. Un-structured problems on the other hand have multiple solutions, may require the learner to choose between several competing internal representations and may require the learner to question several underlying assumptions about what model or representation is applicable in the given situation. These problems are those that typically require significant 'vertical' transfer.

One of the commonly used problem-solving strategies is case-reuse. Jonassen (Jonassen, 2003) describes two kinds of case-reuse strategies often used by learners. The first strategy involves recalling the most similar case from memory and applying it with little or no modification to solve a new problem. This strategy is analogous to what we call 'horizontal' transfer. The second strategy involves using inductive reasoning to construct an internal representation by looking across various cases and later using this representation to solve a new problem. This strategy is analogous to what we call 'vertical' transfer.

Some Caveats

When we distinguish between horizontal and vertical transfer, there are at least a few caveats that we must bear in mind. First, the two transfer process although distinct, and in our opinion fundamentally different from each other, are not mutually exclusive in any way. A given problem scenario when examined closely might require a learner to engage in both kinds of transfer processes. Indeed, Schwartz, Bransford and Sears (Schwartz *et al*., 2005) argue that we must prepare learners to engage in both kinds of transfer rather than one at the expense of the other. They point out that there is indeed value in developing 'efficiency' or 'horizontal' transfer because it frees up the mental resources that allow the mind to focus on other efforts such as being more innovative in other ways. Similarly, diSessa and Wagner (diSessa & Wagner, 2005) point out that most traditional assessments focus on 'Class A' rather than 'Class C' transfer using "few minute, little-or-no-learning" transfer texts. They however do not devalue the use of these tests and state that indeed in some situations such tests can be useful. As researchers' however, it is useful to focus on various types of transfer processes – ‘Class A’ through ‘Class C’. Therefore transfer researchers appear to converge on the consensus that it is important to value both kinds transfer processes.

Second, we believe that there is often no unique definition one can apply to identify whether a particular process involves ‘horizontal’ or ‘vertical’ transfer. If a learner already possesses a well developed knowledge structure such as a “well prepared” coordination class, (diSessa & Wagner, 2005)then from that learner’s perspective a particular task might require only ‘horizontal’ transfer, i.e. applying this well prepared knowledge structure in the present scenario. However, a different learner who does not possess this mental model or internal representation may need to construct one ‘on the fly’ to solve the particular problem. Therefore, this learner has to engage in ‘vertical’ transfer to solve the same problem. This criterion essentially distinguishes between experts and novices. A particular task that might be perceived as ‘horizontal’ transfer by an expert might in fact be perceived as ‘vertical’ transfer from a novice’s perspective. Therefore, any distinction that we attempt to make between the two kinds of transfer must be tied to a particular perspective. In keeping with the learner-centered perspective, we believe it is most useful to view transfer processes from the perspective of the learners who engage in them rather than from a researcher’s perspective.

Finally, it is important to recognize that the distinction between horizontal and vertical transfer depends upon features of the overall learning context. These contextual features may include but are not limited to learners’ or teachers’ expectations and culture of a given situation. For instance, in a mathematics course that focuses on learning how to solve quadratic equations any problem that has a real world connection may be perceived as requiring ‘vertical’ transfer. The same problem, however, in a physics course that routinely expects its students to solve ‘word’ problems might be seen as a regular ‘plug-and-chug’ problem that requires only ‘horizontal’ transfer.

The distinction between horizontal and vertical transfer as we shall see later in this chapter has provided us with a useful theoretical framework to analyze our results, identify problems and hypothesize possible remedies. However, it is important to point out that for all of the aforementioned reasons the framework is not rigid. The distinctions between whether a particular process is categorized as ‘horizontal’ or ‘vertical’ transfer often depends upon the perspective of the learner and researcher and several other contextual factors of the problem scenario.

Research Studies: Transfer from Mathematics to Physics

Prior research on transfer from algebra to physics (Bassok & Holyoak, 1989) found transfer asymmetry between these two domain areas. Most students who learned algebra could apply their knowledge to an isomorphic physics problem; however, very few of the students who learned physics could apply their knowledge to the isomorphic algebra problem. The study appears to highlight the effect of contextual factors in transfer of learning. Learning to apply mathematics in a physics context did not prepare students to solve a more abstract mathematics problem in which the physics context was ‘stripped away.’ We can interpret the results to imply that the physics course did not adequately prepare students to construct new problem solving mental models or schema in contexts other than the physics. The students were unable to engage in vertical transfer. While the Bassok and Holyoak study showed positive transfer from algebra to physics, most physics problems use more than algebra skills. Therefore, we sought to investigate transfer from two other areas of mathematics: calculus and trigonometry.

Transfer from Calculus to Physics

In most U.S. universities, calculus and physics are taught as two separate subjects in their respective departments. Students are usually required to take at least one calculus course prior to taking physics. Integrated curricula have been developed and were found useful in teaching calculus and physics. (Dunn & Barbanel, 2000) Yeats and Hundhausen (Yeatts & Hundhausen, 1992) who used their own experiences in talking about the difficulties -- "notation and symbolism," "the distraction factor" and "compartmentalization of knowledge" – that students have when transferring their knowledge between calculus and physics, also provide some recommendations. However, unlike the integrated curriculum developed by Dunn and Barbanel, (Dunn & Barbanel, 2000) calculus and physics are taught as separate subjects in most universities.

This study focused on how students retained and transferred the knowledge from their calculus course when solving problems in their physics course. We conducted semi-structured one-on-one think-aloud interviews to assess how students transfer their calculus knowledge in a physics context. 'Horizontal' transfer was explored through interviews in which students were asked to solve physics problems that were similar to their homework or exam problems and required use of simple integration or differentiation. We deemed these problems to involve 'horizontal' transfer because from our (i.e. the researchers') perspective the problems did not require students to construct or to even choose between competing schemas or mental models to solve the problem. Interviewees were asked to solve four sets containing two problems each. Each set consisted of a physics problem and an isomorphic calculus problem that utilized the same calculus concept. The goal was to identify the extent to which students would connect the two problems. The problems also provided a context within which to discuss the overall connections between physics and calculus as seen from the students' perspectives.

'Vertical' transfer was assessed using think-aloud interviews in which students were asked two kinds of problems -- 'compare and contrast' problems and jeopardy problems. The 'compare and contrast' problem presented situations in which interviewees would use "integration" instead of "summation." The 'Jeopardy' problems presented interviewees with an intermediate step in the form of a mathematical integration and asked students to come up with a physical scenario relevant to the integral provided. Both of these problems were non-traditional and required students to engage in 'vertical' transfer in several ways. Unlike 'end-of-chapter' problems, the students could not apply a pre-constructed schema or mental model to solve these problems. Because these problems were unfamiliar to students, they had to construct a schema or mental model on the spot to solve these problems. Thus, these problems provided a useful context in which to examine vertical transfer by the students.

In the 'compare and contrast' problem, students were provided with two problem situations such as the ones shown in Figure 2.

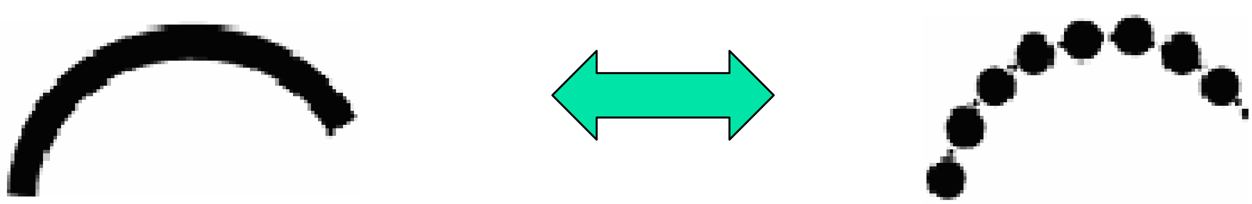

Figure 2: Contrasting two situations. Students are asked to compare the way in which one would find the electric field due to each of the charge distributions shown. The one on the left can be solved using integration while the one of the left requires point wise summation

They were first asked which of the situations would require the use of integration and why. The goal of this type of problem was to examine whether students could transition between two internal representations that are typically used to solve these kinds of problems. One internal representation involves point-wise summation or superposition. The other internal representation involves integration. Learners who productively engage in 'vertical' transfer are typically able to transition between different internal representations depending upon the external representation of the problem.

In 'Jeopardy' problems students were provided with a mathematical expression that included integration as well as some other symbols. An example is shown below.

$$\frac{\mu_0 \int_0^R J(r)\cdot(2\pi r dr)}{2\pi R} \qquad (1)$$

Students were asked to describe the physical problem situation in which would they encounter the expression shown. The goal of this problem was to examine the process by which students deconstructed the external representation provided and reconstruct it in the form of a physical situation. Jeopardy problems have been used by others. Van Heuvelen and Maloney (Van Heuvelen & Maloney, 1999) point out the Jeopardy problems help students prevent the use of typical 'plug-and-chug' methods because they help require students to "give meaning to symbols in an equation" and to "translate between representations in a more robust manner." Learners who productively engage in 'vertical' transfer are typically able to deconstruct an external representation and reconstruct it in a form that matches their internal representation, which in this case is the physical situation corresponding to the problem. Both the 'compare and contrast' tasks as well as 'Jeopardy' problems are by any standards, difficult problems. The focus here was not on learner performance but on the process. Examining how learners approached these problems provided us insights into vertical transfer process.

Our results from examining 'horizontal' transfer while solving 'end-of-chapter' problems indicates that students typically do have an adequate calculus knowledge and skills required for solving end-of-chapter physics problems. Most student difficulties focused around setting up calculus-based physics problems. These difficulties included deciding the appropriate variable and limits of integration. Students often tended to use oversimplified algebraic relationships to avoid using calculus because they do not understand the underlying assumptions of the relationships. It is worth pointing out that when presenting students with 'end-of-chapter' problems we had assumed that the problems would involve 'horizontal' transfer and therefore be perceived as relatively straightforward by the students. It appears however that this was not the case with most students. This observation underscores the caveat that we had mentioned earlier: Whether a given task involves 'horizontal' or 'vertical' transfer depends upon the perspective of the 'actor' or student. What may be perceived as 'horizontal' transfer by the expert may in fact involve 'vertical' transfer from a researcher's point of view.

Finally, when asked what would help them in solving physics problems, most students said they would prefer more application-oriented problems in their calculus course and better scaffolding to solve physics problems. Students also seem to believe that a focus on conceptual understanding and concurrent teaching of calculus and physics would facilitate their application of calculus in physics.

Transfer from Trigonometry to Physics

The second study that we describe here concerns transfer from trigonometry to physics. We measured conceptual understanding in trigonometry in terms of students' abilities to use multiple models or representations of trigonometric relationships. The three models generally used in trigonometry to define trigonometric relationships are in terms of 'right triangles,' the 'unit circle' or as abstract 'functions'.

At the first ('right triangle') level, the use of trigonometric relationships is limited to solving for various features of a right triangle. Students thinking at this level define the basic trigonometric entities in terms of ratios of sides of a right triangle. Students thinking at the geometric level are also able to think abstractly and solve triangles to obtain sides and angles that are labeled with variables. At the second ('unit circle') level, trigonometric relationships are defined in terms of points on a circle. Student thinking at this level is still geometric in nature, but it is different from the first ('right triangle') level because it involves the use of circles and horizontal and vertical projections of a point moving along a circle rather than a triangle. Finally, at the third ('function') level, trigonometric relationships are defined in terms of abstract mathematical function. Thinking at this level is in our view substantially different from the first two levels. At the function level, students must be able to think of trigonometric relationships as divorced from their geometric underpinnings. Thinking at the function level involves extracting physical variables from a given graph or mathematical expression that contains either concrete numbers or variables.

Trigonometric concepts can be understood and applied using different models by the same students depending upon the problem context. While the right triangle model seems to be the easiest and most common for students to learn, our interviews have found that different students can learn the different models in different orders (even within the same class) and that even students who understand multiple models have a great deal of difficulty moving between models but tend to use just one model for each single problem. (Ozimek, 2004; Verbych, 2005)

Students in this study were enrolled in an algebra-based physics course. We focused on those students who had previously taken a trigonometry course at Kansas State University rather than elsewhere. We used two assessments to measure transfer of learning. First, student learning in trigonometry was assessed using online trigonometry homework assignments. Performance was measured by the score divided by the number of attempts taken to achieve that score. Second, transfer to physics was assessed using a multiple choice inventory which contained 18 items that were organized into three groups – one for each model. Each group contained ‘abstract’ questions that tested trigonometry concepts devoid of a physics context paired with ‘contextual’ questions that required students to apply trigonometry concepts in a physics context. The inventory was administered twice to the students: on the first day of class and again after students had completed the relevant material in class.

These assessments allowed us to analyze transfer from trigonometry to physics from both the ‘traditional’ perspective as well as a couple of contemporary perspectives. A different metric was used to assess transfer of learning from each perspective. In our quantitative study we decided that an appropriate metric for the possible existence of transfer was correlation of performance on task T_1 and a subsequent task T_2. While correlation does not imply causality or transfer, it at least indicates the possibility of the existence of transfer. In other words, while a statistically significant correlation does not imply transfer, the lack of a statistically significant correlation between performance on one task T_1 and a performance on a subsequent task T_2 does indicate the lack of transfer.

In one perspective, that we call the ‘traditional’ perspective, transfer is measured as the ability to apply knowledge learned in a prior situation to a new situation. Researchers seek evidence that the students have been able to transfer the pre-defined concept from a context in which the concept was initially learned to a different context. From this perspective, transfer is a static or passive process. Either a student can transfer or a student cannot.

It is important to note that by labeling this perspective as ‘traditional’ we do not intend to imply that it is somehow incorrect or irrelevant in any way. We simply imply that it is incomplete. There are many instances, such as when we want students to learn a simple procedure or skill, when this perspective of measuring transfer can be worthwhile, i.e. either a student has acquired a transferable skill or she has not. However, we believe that if we intend gain insights into complex cognitive processes such as ill-structured problem solving, this perspective may yield incomplete information. Therefore, the ‘traditional’ perspective while useful in some cases may not be productive in other cases.

How does the ‘traditional’ perspective map onto our theoretical framework of ‘horizontal’ versus ‘vertical’ transfer discussed earlier? We believe that adopting a ‘traditional’ perspective to examine transfer is equivalent to seeking application of well-prepared knowledge in new situations. In other words, the ‘traditional’ perspective subsumes that the learner possesses a well prepared knowledge structure, which she should easily be able to activate in a new context. We believe adopting a ‘traditional’ perspective is in many ways equivalent to examining ‘horizontal’ transfer.

As per this perspective, transfer can be measured by comparing performance in the new situation (physics) with performance in the situation in which the knowledge was learned (trigonometry). In our study we assessed transfer in terms of correlations between students' online trigonometry scores with their scores on the 'contextual' physics problems on the pre- and post-instruction surveys for the same model. The rationale was that the physics questions provided a new problem context within which to examine transfer of the trigonometry mathematical concepts learned in the previous course.

In addition to the 'traditional' perspective of transfer described above, two other perspectives were also employed to assess transfer. Taken together we label these the 'contemporary' perspectives. Clearly, other researchers (Beach, 1999; Greeno *et al*., 1993) have also articulated other contemporary perspectives that we do not utilize in our study. Rather we focus on two perspectives -- one by Bransford and Schwartz (Bransford & Schwartz, 1999) and another by. (Lobato, 2003)

Bransford and Schwartz (Bransford & Schwartz, 1999) provide a contemporary perspective of transfer called "preparation for future learning" (PFL). The focus is on whether the initial learning helps students *learn* to solve problems in the new situations with the opportunity to utilize resources (i.e. texts, colleagues, feedback) they may have had available during the initial learning situation. In this study we examined whether students' learning in a trigonometry course prepared them to learn physics. Since students taking our surveys were not permitted to use resources, a way to measure transfer from the PFL perspective is by looking at each student's gain in scores on the physics (contextual) survey questions. These gains serve as a measure of *learning* that occurs during the physics course. The gain on the contextual questions was correlated with the online trigonometry homework assignments and the pre-instruction survey mathematics (abstract) question scores. The rationale for using *gains* is that they are a measure of *learning* in the physics context. Thus, using gains to measure transfer is consistent with the PFL perspective, which views transfer as the ability to learn in the new context. To obtain a deeper insight into transfer of learning from the PFL perspective, the online trigonometry homework assignments and the pre-instruction survey mathematics (abstract) questions were also categorized into the respective models.

Lobato (Lobato, 1996) conceives transfer as the *personal construction of similarities between activities*. She examines transfer by looking at the nature of situations and the similarities people construct across situations. Evidence for transfer is gathered by scrutinizing a given activity for any indication of influence from previous activities. In Lobato's 'Actor-Oriented Transfer' (AOT) perspective, evidence of transfer is found when students create 'relations of similarity' between two situations, i.e. when they notice that two situations are similar in some way. (Lobato, 2003) Therefore, to examine transfer of learning from the actor–oriented perspective, we examined correlations between scores on the 'abstract' (mathematics) and isomorphic 'contextual' (physics) questions using the same model on the same (pre- and post-instruction) survey. The rationale for using correlations between 'abstract' and 'contextual' questions is that the degree of correlation between the scores on an 'abstract' (mathematics) problem and an isomorphic 'contextual' (physics) problem is a measure of the similarities perceived by students between these two problems. The fact that students' performance on two questions is significantly correlated implies that these questions have something similar about them from the students' perspectives. This student-centered notion of perceived similarity is considered sufficient evidence of transfer.

How do the aforementioned ‘contemporary’ perspectives (PFL and AOT) of transfer map onto our theoretical framework of ‘horizontal’ versus ‘vertical’ transfer discussed earlier? Both the PFL and AOT perspectives focus on the dynamics of the transfer process rather than the outcome alone. They do not subsume the existence of a knowledge structure in the learner’s mind rather they focus on the process by which learners construct such a structure in a new situation. Therefore, we believe adopting either of these ‘contemporary’ perspectives is in many ways equivalent to examining ‘vertical’ transfer.

Table 1 summarizes the various perspectives that provided a lens for analyzing our data and understanding the extent to which students transferred their knowledge and conceptual understanding gained in trigonometry to problem solving in physics.

Perspective	**Alignment with Framework**	**Criteria for Transfer**	**Measure of Transfer in this Study**
‘Traditional’	‘Horizontal’	Apply knowledge learned in a prior context to solve a problem in a new context.	Correlation between online trig score per attempt and ‘contextual’ physics score in pre/post survey.
‘Contemporary’ PFL	‘Vertical’	Apply knowledge learned in a prior context to *learn* to solve problems in a new context.	Correlation between online trig score per attempt and *gains* in ‘contextual’ physics questions.
‘Contemporary’ AOT	‘Vertical’	Recognize relations of similarity between the two contexts.	Correlations between ‘abstract’ and ‘contextual’ scores on the *same* survey.

Table 1: Transfer from multiple perspectives, their alignment with our theoretical framework and how they are assessed in this study.

We now discuss the results of our quantitative study from all three perspectives described above. As per the ‘traditional’ perspective, we calculated correlations between average scores on online trigonometry assignments and scores on the ‘contextual’ physics questions (pre- and post-instruction survey) for each model. No statistically significant correlations were found for any of the models for either the pre- or post-instruction survey. Thus, no evidence of transfer was found from the traditional perspective.

As per the PFL perspective, we calculated correlations between the *gains* (post-instruction – pre-instruction) on the physics survey for each model with the scores on online trigonometry assignments for the same model. A statistically significant ($p<0.05$) correlation was found only for the first model (right triangles). Thus, as per the PFL perspective, it appeared that students successfully transferred their learning at the geometric level of triangles but not at the unit circle level or the functional level.

As per Lobato's AOT perspective, we calculated correlations between 'abstract' (mathematics) and 'contextual' (physics) questions on the same survey for both the pre- and post-instruction surveys. Statistically significant correlations were found for the right triangle and functional models on both the pre- and post-instruction surveys. Thus, as per the AOT perspectives it appeared that students were able to dynamically transfer their knowledge from the 'abstract' (mathematics) to the 'contextual' physics questions for right triangles and functions but not for the unit circle.

As expected, the perspective of transfer that we adopted directly influenced whether we found evidence of transfer. We did not find any evidence of transfer from trigonometry to physics as per the 'traditional' perspective. However, when we broadened our perspective we found evidence of transfer as per the 'contemporary' PFL and AOT perspectives. We believe that this observation is not a weakness of our study; rather it underscores the importance of examining transfer from a variety of different perspectives. Transfer was also found to be nonuniform across models. Stronger evidence of transfer was detected for right triangle concepts than for function concepts and none was detected for unit circle concepts. This may not be unrelated to the fact that trigonometry students are generally more comfortable with right triangle concepts.

Table 2 summarizes our results. Each checkmark indicates evidence of transfer from the corresponding perspective and model. These results appear to converge on two conclusions. First, students have most difficulty in transferring the unit circle model to solving problems in physics. This fact may indicate that the learners do not view the unit-circle model as being useful or do not have an adequately developed model that they can use in a new situation. Second, 'vertical' transfer appeared to have occurred more often than 'horizontal' transfer for all but the unit circle model.

Perspective	**Type of Transfer**	**Model**		
		Right Triangle	**Unit Circle**	**Function**
'Traditional'	'Horizontal'			
'Contemporary' PFL	'Vertical'	✓		
'Contemporary' AOT	'Vertical'	✓		✓

Table 2: Multiple perspectives of transfer and their assessment used in this study

At first glance, this result may appear to be contrary to our expectations – after all, isn't 'vertical' transfer more challenging than 'horizontal' transfer? If so, why do we not see more evidence of 'horizontal' transfer than 'vertical' transfer? We believe that there is in fact no contradiction. In presenting our theoretical framework we stated that 'horizontal' transfer involved activating a knowledge structure in a given situation and associating the variables of the problem situation with elements in the knowledge structure. This description of 'horizontal' transfer subsumes that the learner already possesses a knowledge structure that they can appropriately activate whenever necessary. diSessa and Wagner (diSessa & Wagner, 2005) call this type of transfer 'Class A' transfer. They point out that 'Class A' transfer is in fact rather rare because most students do not possess what they call a "well prepared" coordination class that they can activate appropriately in wide range of scenarios. They contrast this kind of transfer with 'Class C' transfer in which "relatively unprepared" learners reuse prior knowledge in a new scenario. 'Class C' transfer is in fact indistinguishable from learning. Therefore, 'horizontal' (or 'Class A') transfer is indeed rare because it requires learners to have first developed an internal knowledge structure that they can activate in a variety of different situations. The inherent difficulty that learners experience as they construct an internal knowledge structure is what makes 'horizontal' transfer rare as observed in this study.

Conclusions

We have presented a theoretical framework that describes transfer of learning in problem solving. This framework builds on our earlier model of transfer (Rebello *et al.*, 2005) in which transfer is the dynamic creation of associations between information read-out by the learner in a new situation and a learner's prior knowledge. Our framework distinguishes between two kinds of transfer processes that we believe, though not mutually exclusive, are fundamentally different from each other. 'Horizontal' transfer involves associations between a learner's well developed internal knowledge structure and new information gathered by the learner. 'Vertical' transfer involves associations between various knowledge elements that result in the creation of a new knowledge structure that is productive in the new situation.

The studies described in this chapter have several implications for educators and educational researchers who are interested in transfer of learning to aid problem solving in semi-structured or unstructured domains. Our results demonstrate that transfer of learning from structured domains such as mathematics to relatively semi-structured or unstructured domains such as physics or engineering must be examined from multiple perspectives of transfer. When viewed from a traditional perspective that focuses primarily on 'horizontal' transfer of a well developed internal knowledge structure, students often appear to fail to transfer what they have learned in one context to solve problems in another context. However, upon expanding our perspective to focus on students' abilities to learn how to solve problems in the new context by building new knowledge structures as in 'vertical' transfer, we are more likely to find evidence of transfer.

Educators have sometimes speculated whether providing students with a structured problem followed by a semi-structured isomorphic problem could increase performance on the latter. Results from our studies indicate that students may recognize that the problems are similar in some ways. However, these constructions of similarity by the students do not necessarily translate into improved performance by the students on problems in the unstructured domain. We found no evidence that providing students with the structured problem will necessarily help them solve the isomorphic semi-structured problem that follows on the same exam.

Many physics and engineering educators often lament that their students do not enter their class with the adequate mathematics preparation. Our results appear to indicate that the main difficulty that students appear to have does not lie in their lack of understanding of mathematics per se, rather it lies in their inability to see how mathematics is appropriately applied to physics problems. Students often do not understand the underlying assumptions and approximations that they might need to make in a physics problem before they apply a particular mathematical strategy. It appears that students do not possess adequately well prepared internal knowledge structures pertaining to solving quantitative physics problems that require the use of mathematics. Their structures are inadequate in that students are often unable to align their internal knowledge structures with the problem information (‘horizontal’ transfer). They are also equally unable to modify or choose between knowledge structures (‘vertical’ transfer). Therefore, they apply mathematical strategies that are inconsistent with the particular situation, for instance, using discrete summation rather than continuous integration in a given problem.

Our research also provides some insights into strategies that students believe might be helpful to them as they transition from mathematics to physics or engineering classes where they apply their mathematics knowledge in relatively semi-structured problems. To adequately prepare them for these classes, mathematics classes that often focus on developing students’ mathematical skills should also provide opportunities for helping students solve contextualized and semi-structured “word” problems. In studying transfer from one mathematics problem to another, Schoenfeld (Schoenfeld, 1985) found that explicit instruction in recognizing similarities improved students’ abilities to transfer ideas in solving novel problems. The students’ requests for increased word problems in calculus may be related to their need for seeing such explicit instruction in recognizing similarities across contexts. Physics courses should facilitate students’ development of their problem-solving skills by helping them learn how to set up semi-structured problems. Both mathematics and physics courses should focus on helping students understand the concepts that underpin the mathematical strategies and equations that they use rather than merely the strategies themselves. Finally, the mathematics and physics courses should be taught in an integrated format, or at least concurrently so that students can be provided adequate opportunities to transfer internal knowledge structures that they have constructed in their mathematics course to solving problems in relatively semi-structured domains such as physics.

Acknowledgements

This work is supported in part by the U.S. National Science Foundation Grants DUE-0206943 and REC-0133621. Opinions expressed in this article are of the authors and not necessarily those of the Foundation.

References Cited

Adams, L., Kasserman, J., Yearwood, A., Perfetto, G. A., Bransford, J. D., & Franks, J. J. (1988). The effects of facts versus problem-oriented acquisition. *Memory and Cognition, 16*, 167-175.

Bassok, M. (1990). Transfer of domain-specific problem-solving procedures. *Journal of Experimental Psychology: Learning*, 522-533.

Bassok, M., & Holyoak, K. (1989). Interdomain transfer between isomorphic topics in algebra and physics. *Journal of Experimental Psychology: Learning*, 153-166.

Beach, K. (1999). Consequential transitions: A sociocultural expedition beyond transfer in education. *Review of Research in Education, 24,* 101-140.

Bransford, J. D., & Schwartz, D. (1999). Rethinking transfer: A simple proposal with multiple implications. *Review of Research in Education, 24*, 61-100.

Broudy, H. S. (1977). Types of knowledge and purposes of education. In C. Anderson, R. J. Spiro & W. E. Montague (Eds.), *Schooling and the acquisition of knowledge*. Hillsdale, NJ: Erlbaum.

Brown, A. L., & Kane, M. J. (1988). Preschool children can learn to transfer: Learning to learn and learning from example. *Cognitive Psychology, 20*, 493-523.

Chen, Z., & Daehler, M. W. (1989). Positive and negative transfer in analogical problem solving. *Cognitive Development, 4*, 327-344.

DiSessa, A. (1988). Knowledge in pieces. In G. Forman & P. B. Pufall (Eds.), *Constructivism in the computer age* (pp. 49-70). Hillsdale, NJ: Lawrence Erlbaum Associates.

DiSessa, A. (1993). Towards an epistemology of physics. *Cognition and Instruction, 10*((2-3)), 105-225.

DiSessa, A. (1998). What changes in conceptual change? *International Journal of Science Education, 20*(10), 1155-1191.

DiSessa, A., & Wagner, J. (2005). What Coordination Has to Say about Transfer. In J. P. Mestre (Ed.), *Transfer of Learning from a Modern Multidisciplinary Perspective*. Greenwich, CT: nformation Age Publishing Inc.

Duncker, K. (1945). On problem solving. *Psychological Monographs, 58*(270).

Dunn, J. W., & Barbanel, J. (2000). One model for an integrated math/physics course focusing on electricity and magnetism and related calculus topics. *American Journal of Physics, 68*, 749.

Engelhardt, P. V., Corpuz, E. G., Ozimek, D. J., & Rebello, N. S. (2003). *The Teaching Experiment - What it is and what it isn't.* Paper presented at the Physics Education Research Conference, 2003, Madison, WI.

Gick, M. L., and Holyoak, K. J. (1980). Analogical Problem Solving. *Cognitive Psychology, 12*, 306-355.

Greeno, J. G., Moore, J. L., & Smith, D. R. (1993). Transfer of situated learning. In D. K. Detterman & R. J. Sternberg (Eds.), *Transfer on trial: Intelligence, cognition and instruction* (pp. 99-167). Norwood, NJ: Ablex.

Hammer, D. (2000). Student Resources for Learning Introductory Physics. *American Journal of Physics - Physics Education Research Supplement, 68*(7), S52-S59.

Hammer, D., & Elby, A. (2002). On the form of a personal epistemology. In P. R. Pintrich & B. K. Hofer (Eds.), *Personal Epistemolgy: The Psychology of Beliefs about Knowledge and Knowing* (pp. 169-190). Mahwah, N.J.: Lawrence Erlbaum.

Jonassen, D. H. (2000). Toward a design theory of problem solving. *Educational Technology: Research & Development,, 48*(4), 63.

Jonassen, D. H. (2003). Using cognitive tools to represent problems. *Journal of Research in Technology in Education, 35*(3), 362-381.

Judd, C. H. (1908). The relation of special training to general intelligence. *Educational Review, 36*(28-42).

Lobato, J. E. (1996). *Transfer Reconceived: How "sameness" is produced in mathematical activity, Ph.D. Dissertation.* Unpublished Ph.D. Dissertation, University of California, Berkeley, Berkeley, CA.

Lobato, J. E. (2003). How Design Experiments Can Inform a Rethinking of Transfer and Vice Versa. *Educational Researcher, 32*(1), 17-20.

McKeough, R. E., Lupart, J., & Marini, A. (1995). *Teaching for transfer: Fostering generalization in learning.* Mahawah, NJ: Erlbaum.

Nisbett, R. E., Fong, G. T., Lehmann, D. R., & Cheng, P. W. (1987). Teaching reasoning. *Science, 238*, 625-630.

Novick, L. (1988). Analogical transfer, problem similarity, and expertise. *Journal of Experimental Psychology: Learning, Memory & Cognition, 14,* 510-520.

Ozimek, D. J. (2004). *Student Learning, Retention and Transfer from Trigonometry to Physics.* Unpublished M.S. Thesis, Kansas State University, Manhattan, KS.

Perfetto, G. A., Bransford, J. D., Franks, J. J. (1983). Constraints on access in a problem solving context. *Memory and Cognition*, *11*, 12-31.

Piaget, J. (1964). Development and Learning. *Journal of Research in Science Teaching*, *2*(3), 176-186.

Pólya, G. (1957). *How to Solve It* (2nd ed.). New York, NY: Doubleday.

Rebello, N. S., Zollman, D. A., Allbaugh, A. R., Engelhardt, P. V., Gray, K. E., Hrepic, Z., *et al*. (2005). Dynamic Transfer: A Perspective from Physics Education Research. In J. P. Mestre (Ed.), *Transfer of Learning from a Modern Multidisciplinary Perspective*. Greenwich, CT: Information Age Publishing Inc.

Redish, E. F. (2004, July 15-25, 2003). *A Theoretical Framework for Physics Education Research: Modeling Student Thinking*. Paper presented at the International School of Physics, "Enrico Fermi", Course CLVI, Varenna, Italy.

Reed, S. K. (1993). A schema-based theory of transfer. In D. K. Detterman & R. J. Sternberg (Eds.), *Transfer on trial: Intelligence, Cognition and Instruction* (pp. 39-67). Norwood, NJ: Ablex.

Reed, S. K., Ernst, G. W., & Banerji, R. (1974). The role of analogy in transfer between similar problem states. *Cognitive Psychology*, *6*, 436-450.

Salomon, G., & Perkins, D. N. (1989). Rocky roads to transfer: Rethinking mechanisms of a neglected phenomenon. *Educational Psychologist, 24*(2), 113-142.

Schoenfeld, A. (1985). *Mathematical Problem Solving,* . Orlando: Academic Press.

Schwartz, D., Bransford, J. D., & Sears, D. (2005). Efficiency and Innovation in Transfer. In J. P. Mestre (Ed.), *Transfer of Learning from a Modern Multidisciplinary Perspective*. Greenwich, CT: Information Age Publishing.

Singley, K., & Anderson, J. R. (1989). *The Transfer of Cognitive Skill*. Cambridge, MA: Harvard University Press.

Steffe, L. P., & Thompson, P. W. (2000). Teaching experiment methodology: Underlying principles and essential elements. In R. Lesh & A. E. Kelly (Eds.), *Research design in mathematics and science education.* (pp. 267-307). Hillsdale, NJ: Erlbaum.

Throndike, E. L., & Woodworth, R. S. (1901). The influence of improvement in one mental function upon the efficacy of other functions. *Psychological Review, 8*, 247-261.

Van Heuvelen, A., & Maloney, D. P. (1999). Playing Physics Jeopardy. *American Journal of Physics, 67*(3), 252-256.

Verbych, E. (2005). *Student work and success using online homework in a trigonometry class.* Unpublished M.S. Thesis, Kansas State University, Manhattan, KS.

Wertheimer, M. (1959). *Productive Thinking*. New York, NY: Harper & Row.

Yeatts, F. R., & Hundhausen, J. R. (1992). Calculus and physics: Challenges at the interface. *American Journal of Physics, 60*, 716.