TRANSFER OF LEARNING FROM COLLEGE CALCULUS TO PHYSICS COURSES

This research investigated students' transfer of learning from calculus courses to an introductory physics course. We used semi-structured think aloud interviews to assess the extent to which students transfer their calculus knowledge when solving problems in a physics course. Results indicate that students do transfer their knowledge from calculus class to physics class. However, during the transfer process they needed specific scaffolding to connect the calculus knowledge with the physics problem.

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Background and Introduction

Transfer of learning has often been referred to as the central goal of education. (McKeough, Lupart & Marini, 1995) In this study we focus specifically on transfer from calculus to physics. Since calculus is a necessary preparation course for calculus-based physics, sometimes called engineering physics, how students transfer their knowledge learned in calculus can be critical to their learning in physics. Therefore, assessing the transfer from calculus to physics is the goal of this study. To realize this goal, we address the following research questions

- To what extent do students retain and transfer their calculus knowledge when solving problems in introductory physics?
- What difficulties pertaining to the transfer of calculus do students have while solving these problems?
- What strategies may help students overcome these difficulties?

Relevant Literature

Transfer is often defined as the ability to apply what has been learned in one context to a new context (e.g. Byrnes, 1996). To assess transfer, researchers have often used one-shot assessments such as performance on tests and examinations. Contemporary perspectives describe transfer as a dynamic construction of associations between the two contexts mediated by several factors. (Rebello et al., 2005) For this study graduated prompting is considered an effective way to assess transfer and the factors that control it. (Newmann, 1989)

When assessing transfer from math to physics, Bassok (Bassok & Holyoak, 1989) found transfer asymmetry between algebra and physics. Most students who learned algebra only could apply the algebra knowledge to the isomorphic physics problem based upon the targeted algebra concept; however, very few of the students who learned physics alone could apply their physics knowledge to the isomorphic algebra problem. Ozimek

(Ozimek, 2004) examined the retention and transfer from trigonometry to physics. From the traditional view of transfer, he found no evidence of transfer. However, from the contemporary perspectives, he found that students do transfer what they learned in trigonometry to physics. Integrated curricula have been developed and found useful to teach calculus and physics. (e.g. Dunn & Barbanel, 2000) Yeatts and Hundhausen (Yeatts & Hundhausen, 1992) used their own experience in talking about the difficulties – "notation and symbolism," "the distraction factor" and "compartmentalization of knowledge" – that students have when transferring their knowledge between calculus and physics and provide some recommendations.

Unlike the curriculum developed by Dunn, (Dunn & Barbanel, 2000) in most U.S. universities calculus and physics are taught as two separate subjects in different departments. Students are usually required to take calculus before taking physics. This study focused on how students retained and transferred their calculus knowledge when solving physics problems.

Methodology

Because transfer is a dynamic process from the contemporary perspective, we conducted semi-structured, one-on-one think aloud interviews to assess how students transfer their calculus knowledge in a physics context.

Engineering Physics (EP) II students who enrolled in Fall 2004 and Spring 2005 at a large mid-western land-grant university participated in this study. EP is a calculus-based physics course, two semesters long. Approximately 80% of students enrolled in EP are engineering majors. We chose EPII because it requires a significant application of differential and integral calculus. Students typically enroll in at least one calculus course before they take any EP course.

Each participant was interviewed over two sessions; each lasting about one hour. The interviewee was left alone when solving the assigned problem. Upon completion, we asked interviewees to explain what they had written down and encouraged them to verbalize their thinking process. We also asked them to describe any difficulties they had when solving the problem. To complete the interview general questions about calculus background and application of their calculus knowledge in physics were asked.

We asked the interviewee to solve physics problems which were similar to their homework or exam problems and required use of simple integration or differentiation. The four physics problems we used were: 1) E field caused by a half-circle charge distribution, 2) electric potential caused by changing E field, 3) B field caused by a non-constant current distribution and 4) induced current caused by moving of the loop in a changing magnetic field. The study was completed in two phases.

Phase I

Eight male volunteers were paid to participate in the interviews. Interviewees were asked to solve four sets of two problems each. Each set consisted of a physics problem and an isomorphic calculus problem that utilized the same calculus concept. The goal was to identify the extent to which students would connect the two problems. The problems also provided a context within which to discuss the overall connections between physics and calculus, as seen from the students' perspective.

Phase II

Five male and three female paid volunteers participated in the interviews. Based on our results of Phase I (discussed later) we asked the interviewees to solve a physics problem and explored the origin of their difficulties. After we had asked students to describe how they solved the problem, we presented them with variations of the problems that they had just solved. These variations explored the criteria based on which interviewees used "integration" instead of "summation" and we asked them to solve three variations of the physics problems below.

Variation I: As the variation of "E field caused by a half-circle charge distribution" question, we asked interviewees whether they would use the same method if there were several point charges instead of an arc-shaped charge distribution (Figure 1)

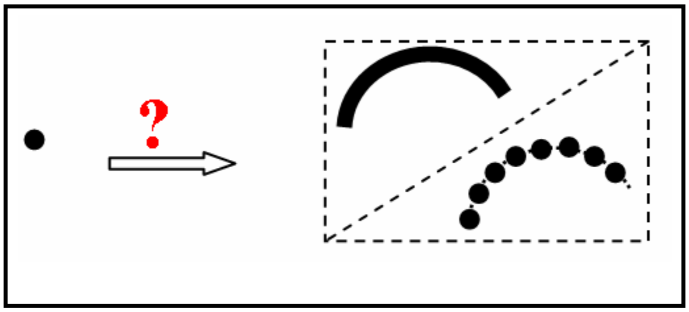

Figure 1: Variation I

Variation II: As the variation of "Magnetic field caused by a non-constant current distribution," we asked what would be the difference if we changed the constant current distribution into a few very thin layers of current

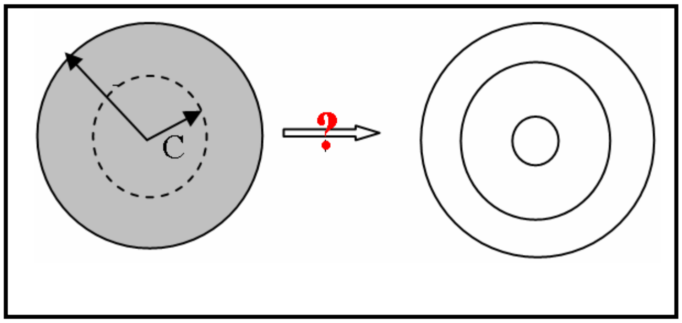

Figure 2: Variation II

Variation III: As a variation of "Induced current caused by moving of the loop in a changing magnetic field" problem, we asked the students to consider what would be the difference for the four cases shown below (Figure 3), with the very small loops.

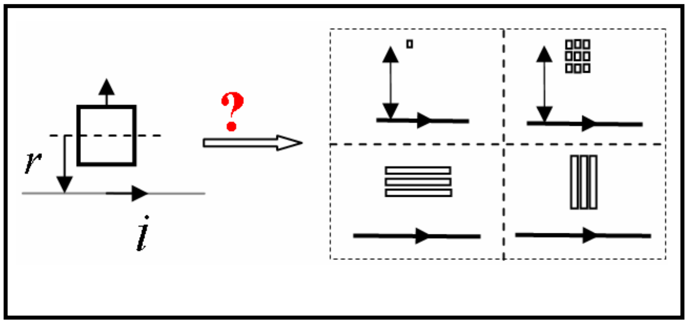

Figure 3: Variation III

Results

Phase I

The following themes emerged from the analysis of categories of description identified in Phase I.

Self-confidence in calculus knowledge retention

All interviewees had taken Calculus I and II before taking EP II. Three out of eight interviewees had positive experiences in their calculus class, three had negative experiences and the other two were neutral. However, all of the interviewees were satisfied and confident of their calculus knowledge.

Realization that calculus is required in physics

All of the students realized that physics and mathematics were inextricably linked. As one student commented: "Physics talks about why to solve it, math talks about how to solve it." They also realized that they needed calculus knowledge to solve the physics problems. Seven out of eight interviewees thought their calculus knowledge was sufficient for them to use in physics class. But, students were evenly split when asked whether it would be possible to set up the physics problems without calculus. Furthermore, when asked to compare the problems, only the students who successfully solved the physics problem could see the similarities in the problems. Solving the calculus problem did not help interviewees to solve the physics problem.

Lack of confidence in setting-up physics problems

All of the students had seen physics problems similar to the interview question before. However, none of them were confident of their physics solution strategy. Students were particularly unclear about the criteria that determined whether calculus should be applied in a given problem.

Phase II

In Phase I we had identified that students' difficulties in problem solving in EPII were mainly concerned with setting up the calculus-based physics problem rather than with the calculus per se. In Phase II, we explored these difficulties using the three problem variations earlier described, which helped us explore students' understanding of the criteria that determined whether integral calculus was applicable to the physics problems. The analysis generated a set of results which are presented as answers to the following five questions.

When do you use integration in physics problems?

Seven out of eight interviewees appropriately used integration to solve the physics problems, while one student did not use calculus even after several hints. When the students that used calculus were asked about the criteria they applied to decide why calculus was applicable to the problem, four out of seven interviewees said the problems were similar to the examples they had seen in the text; however, they could not explain why they used integration. Three out of seven interviewees had a rough idea as to why they needed to use integration in terms of adding up the infinitesimally small elements: *"...you can not add up an infinite number…then I used integral…"* Interviewees commented that they had not received any specific formal instruction on this topic. Almost all students, even those who did not articulate the criteria above, could solve the Variation I problem described earlier. Very few interviewees could correctly solve the Variation II and III problems.

What are the difficulties when applying integrals?

The following themes emerged in students' responses when asked about difficulties in applying integrals:

Determining the variable of integration: All interviewees complained that they had difficulty figuring out what was the "real" variable that needed to be integrated or differentiated. Some commented that *"…all constants (variables), I do not know what I should integrate although I know how to integrate…"* Interviewees who figured it out, stated that they *"got it from both calculus and physics, just look for whatever is changing…"*

Deciding the limits of integration: Most interviewees had difficulties in setting up the limits of integration. About half ascribed the difficulties to the physics class. One remarked that it has*"…not really to do with my math class…I know how to integrate it, but it is just figuring out what to integrate."* Others felt that the calculus class was to blame *"because the physics concept is pretty simple…, it is writing an equation for what I understood that is hard…"*

Do students prefer to use calculus or a pre-derived algebraic relationship?

Most interviewees tended to use pre-derived formulae rather than using calculus to derive the formulae from first principles. This tendency led to several difficulties. For instance, they would directly write:

$$E = \frac{\mu_0 i}{2\pi r} \qquad (1)$$

instead of using

$$\oint \vec{E} ds = \mu_0 i_{enclosed} \qquad (2)$$

and then apply it to derive the algebraic relationship. However, when they used the algebraic formula, they were not aware of the conditions in which the formula was applicable. When we asked interviewees why they preferred using algebra rather than calculus they remarked that, typical reasoning as, "…*more confidently use algebra expression to go straight rather than understand this (calculus)…*"

Does using calculus in physics need understanding or just 'plug and chug'?

Six out of eight interviewees felt that applying calculus in physics is more or less 'plug and chug.' They said: *"I do not need to understand it, just how to do it. And I was doing good this way in calculus…"*

Two out of eight interviewees believed that they needed to understand calculus or "*I will be confused*".

What should we do to facilitate transfer between calculus and physics?

When students were asked about how their calculus or physics classes could be reformed to facilitate their learning, the following themes emerged.

<u>More 'word' problems in calculus</u>: Students would prefer more application-oriented problems in calculus to prepare them for future applications.

<u>Learning how to set-up physics problems</u>: Students would prefer more step-by-step scaffolding to help them solve problems in physics.

<u>Focus on understanding:</u> Students would prefer a focus on understanding rather than on memorizing equations *"even in calculus, I had to understand why the differentiation of* s^2 *equal to 2s…", "why integration and differentiation works."*

<u>Course sequencing</u>: Students would prefer to take calculus and physics concurrently because *"you will have more opportunities to use and understand it..."*

Conclusions

Our results indicate that students believe that for the most part their calculus class has provided them with adequate knowledge and skills required for physics. Students acknowledged they had difficulties in setting up calculus-based physics problems. These difficulties include: deciding the appropriate variable and limits of integration; not being clear about the criteria which determine whether calculus is applicable in a given physics problem; and tended to use oversimplified algebraic relationships to avoid using calculus because they do not understand the underlying assumptions of the relationships.

Students would prefer more application oriented problems in their calculus course and better scaffolding to help solve physics problems. Students also seem to believe that a focus on conceptual understanding and concurrent teaching of calculus physics would facilitate their application of calculus knowledge in physics.

Our interview results represented a small portion of students taking these courses, but were helpful in identifying the aforementioned issues. In the next phase we will expand our study to design a quantitative investigation of students' performance on calculus-based physics problems on exams.

References

Bassok, M., & Holyoak, K. (1989). Interdomain transfer between isomorphic topics in algebra and physics. *Journal of Experimental Psychology: Learning*, 153-166.

Byrnes, J. P. (1996). *Cognitive Development and Learning in Instructional Contexts*. Boston, MA: Allyn and Bacon.

Dunn, J. W., & Barbanel, J. (2000). One model for an integrated math/physics course focusing on electricity and magnetism and related calculus topics. *American Journal of Physics, 68*, 749.

Lobato, J. E. (2003). How Design Experiments Can Inform a Rethinking of Transfer and Vice Versa. *Educational Researcher, 32*(1), 17-20.

Marton, F. (1986). Phenomenography - a research approach to investigating different understanding of reality. *Journal of Thought, 21*, 29-39.

McKeough, R. E., Lupart, J., & Marini, A. (1995). *Teaching for transfer: Fostering generalization in learning.* Mahawah, NJ: Erlbaum.

Newmann, D., Griffin. P., Cole, M. (1989). *The Construction Zone: Working for Cognitive Change in School*. New York: Cambridge University Press.

Ozimek, D. J. (2004). *Student Learning, Retention and Transfer from Trigonometry to Physics.* Kansas State University, Manhattan, KS.

Rebello, N. S., Zollman, D. A., Allbaugh, A. R., Engelhardt, P. V., Gray, K. E., Hrepic, Z., et al. (2005). Dynamic Transfer: A Perspective from Physics Education Research. In J. P. Mestre (Ed.), *Transfer of Learning from a Modern Multidisciplinary Perspective*. Greenwich, CT: Information Age Publishing Inc.

Yeatts, F. R., & Hundhausen, J. R. (1992). Calculus and physics: Challenges at the interface. *American Journal of Physics, 60*, 716.