Visualizing Motion in Potential Wells*

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Abstract: The concept of potential energy diagrams is of fundamental importance in the study of quantum physics. Yet, students are rarely exposed to this powerful alternative description in introductory classes and, thus, have difficulty comprehending its significance when they encounter it in beginning level quantum courses. We describe a learning unit that incorporates a sequence of computer interfaced experiments using dynamics or air track systems. This unit is designed to make the learning of potential energy diagrams less abstract. Students begin by constructing the harmonic or square well potential diagrams using either the velocity data and assuming conservation of energy or the force-displacement graph for the elastic interaction of an object constrained by springs or bouncing off springy blocks. Then, they investigate the motion of a rider magnet interacting with a configuration of field magnets, directly and plot the potential energy diagrams using a magnetic field sensor. The ease of measurement allows exploring the motion in a large variety of potential shapes in a short duration class.

I. Introduction

The concept of potential energy diagrams is of fundamental importance to teaching of physics. In many situations, instead of Newton's law, energy considerations are used as the basis for analyzing the behavior of an object interacting with a given system. The procedure is to consider how the potential energy of the object varies as a function of distance. Given this information and the total energy of the object, all other details of behavior can be deduced. This method is adopted in quantum physics for studying the motion of microscopic particles. Students in beginning level mechanics courses, however, are rarely exposed to this powerful alternative description of motion. Thus they frequently have difficulty comprehending the significance and power of potential energy functions when they first encounter them in introductory quantum mechanics.

We have attempted to remedy this situation by creating a set of experiments and a teaching unit designed to provide beginning level students with an early exposure to the idea of potential energy diagrams. Familiar mechanical systems are used to create potential shapes akin to those encountered in models of atomic and nuclear interactions. Observing and interpreting motion in these potential wells on the one hand and establishing the relationship between the operative forces and the potential energy on the other, provides a straightforward understanding of an otherwise abstract concept.

A survey of literature shows that very few earlier attempts have set up experiments to study motion in potential wells[1,2,3]. The most comprehensive effort using this approach is in the work of Saraf et al[4]. The essential idea in all these laboratory investigations is to place magnets along an air track to create a variety of potential barriers and wells. The previous developers used a photo-interrupt and timing device to measure directly the velocity of a glider at a number of locations along its path. Then Assuming conservation of energy, their students determined the potential energy as a function of distance. Sometimes for the sake of economy a single sensor was employed. Then, the experiment had to be repeated a large number of times with the sensor placed at different locations. This procedure made the experiment rather tedious, and variations in initial conditions usually led to a relatively large scatter in the data.

Prior to the development of microcomputer-based laboratory equipment designing an experiment that could yield the velocities at different locations along the path in a single run was not trivial. Thus, all such earlier work employed dedicated transducers and computer programs not commonly available off the shelf. For instance, Eckstein[3,5] used a specially designed motion detector which is essentially a Mylar strip crossed by regularly marked black stripes. As the object was in motion, the number of stripes passing through two high resolution photocells was counted as a function of time. The students ported these data into a spreadsheet to compute the velocities The procedural problems inherent in these methods make the experiments rather complicated in practice and difficult to execute in a typical undergraduate class.

While magnetic interactions generate very interesting shapes for the potential function, the force field of an arbitrary configuration of magnets can be rather complicated and the motion, difficult to predict. For this reason, we recommend introducing the notion of potential well using the more familiar elastic interactions at the outset. In our approach, students first construct potential wells for a simple physical system such as a cart oscillating along a track constrained by springs or moving freely along a track and bouncing off springy blocks placed at the ends of the track. Once the students have built sufficient competence in identifying the forces and interpreting the corresponding potential energy diagrams in these cases, they explore the motion in more complicated potential shapes generated by configurations of magnets.

We have designed our experiments using transducers and data acquisition systems readily available from commercial suppliers. However, the setups and the measurement procedures incorporate several novel features. In the following sections, we provide details of experiment design and outline the learning paths envisaged.

II. Experiments

1. Elastic interactions

We have used the data acquisition system ULI, the program Motion, the Student Force Sensor and the Motion Detector, all available from Vernier Software[6], to record the motion and the force in real time. The experiments use either the dynamics cart or the air track system from PASCO scientific[7]. The advantage of using the dynamics cart is that the setup is relatively inexpensive, involves few adjustments and is more robust for classroom usage, while the advantage of using the air track is that smaller friction means that the crucial assumption of conservation of energy holds better and the motion can be observed over a longer duration of time.

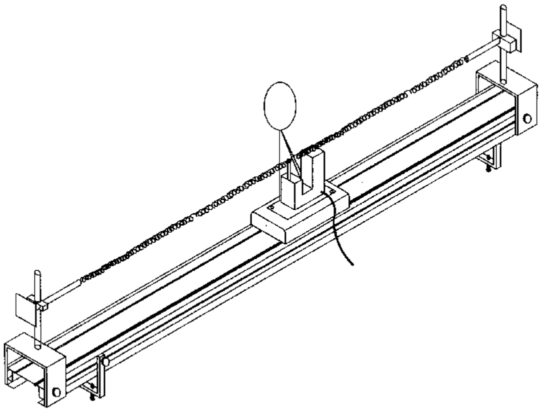

Figure 1. The experimental arrangement for investigating potential energy diagrams using dynamics carts and springs.

a. Spring Oscillator

Dynamics cart system: Fig. 1 gives a schematic diagram of the basic setup with a cart held to the two ends of the track by a set of matched springs. The Student Force Sensor is mounted on the cart using a flat plate with a small post on which the sensor can be screwed. This plate also mounts another post on which a disk is fixed at a suitable height to reflect the signal from the ultrasonic motion detector. Our investigations show that a circular disk works quite well for the purpose. The springs attach on one end to the hooks welded by the manufacturer to the cantilever beam of the force sensor and on the other to specially designed adjustable clamps fixed at the two ends of the track. The end clamps allow for easy alignment of the springs. Care is also taken to eliminate any wobble of the force sensor or the reflector to ensure low-noise data for the position and the force.

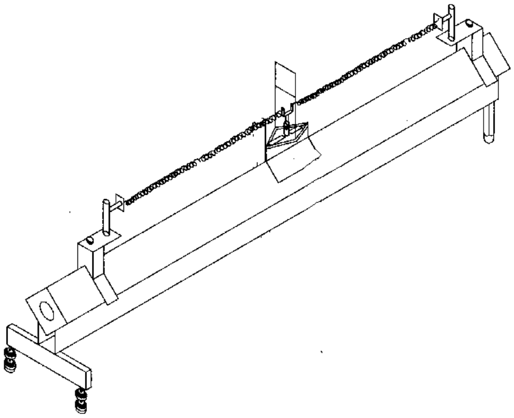

Figure 2. The experimental arrangement for investigating potential energy diagrams using air track gliders and springs.

Air track system: Because of the added weight the glider can not float easily if the commercially available students' force sensor is mounted on it. An excellent and relatively inexpensive alternative is to use the strain gage force sensor kit[8] available from Vernier and fabricate a much lighter force sensor. This kit contains a set of strain gauge transducers, the circuit diagram and the electrical components for constructing an off-balance bridge for measuring the analog signal. We glued the transducers symmetrically on opposite faces of the free end of a narrow spring iron cantilever beam of approximate width 1 cm and length 10 cm. The other end of the beam was clamped in a metal piece that could be plugged into a socket that exists on the top plate of the glider. A U shaped piece was welded to the free end of this beam. Two small holes were drilled on this piece to hook the springs. The reflecting disk was mounted on the glider using a simple clamp arrangement that could be fitted to the base of the glider. This entire mechanical assembly is sketched in Fig.2. The end posts for attaching the springs were made from an acrylic wedge machined to snap on to the air track. It is important to check that the addition of this hardware does not make the glider off balance or block the flow of air.

An important advantage of fabricating this force sensor has been that the leads of the electrical circuit could be made from extremely light and flexible connecting wire. We suspended this wire from a tall clamp to keep it from rubbing against the surface of the laboratory table. This arrangement overcomes the problems posed by the thick short cable that leads the signal from the manufacturer assembled student force sensor to the computer interface. This cable rubbing against the table introduces much of the friction in the dynamic cart.

Harmonic Potential Well

The data acquisition software records the distance of the cart from the position of the motion detector. A simple transformation using the data manipulation facilities of the software converts this measurement to a displacement about the equilibrium position of the cart. The program also automatically computes the velocities from the raw position data. The students view the displacement-time, force-time graphs and correlate these results with the on-line plots of force-displacement and velocity-displacement curves.

These data are then used to construct the potential energy diagram by using one of several options.

(1) Assuming conservation of energy, the program can be used to compute the kinetic energy as the derived quantity $KE = \frac{1}{2}mv^2$ and the potential energy as $PE = TE - KE$.

(2) The students can fit a linear least square line to the force-displacement graph, relate the slope of the line to the effective spring constant k and then compute the potential energy as $PE = \frac{1}{2}kx^2$.

(3) The potential energy can be found by superposing a grid on the force-displacement graph and estimating the area under the curve.

Fig. 3 displays the potential energy function computed as $PE = TE - KE$ from observations of a spring oscillator on an air track[9]. The best quadratic fit is superposed as the solid line; the spread in the potential energy values indicates the extent of the inevitable energy losses and the limits of validity of the assumption of conservation of energy.

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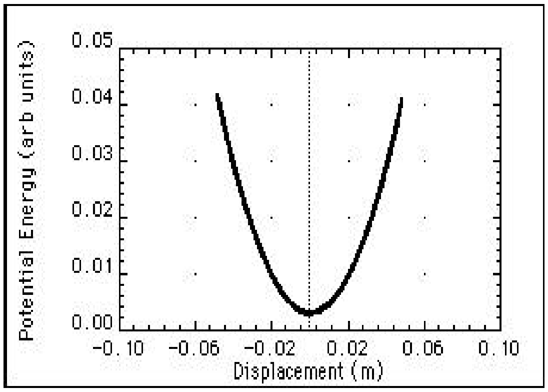


Figure 3: Harmonic Potential Well for a spring oscillator on an air track. The solid line is the best quadratic fit to the potential energy values computed assuming conservation of energy.

b. Bouncing off springy walls

An important quantum mechanical system is a free particle bound in an energy well. A simple analog model of this situation is provided by a cart rolling freely along the track and bouncing off springy blocks placed at the edges (Fig.4).

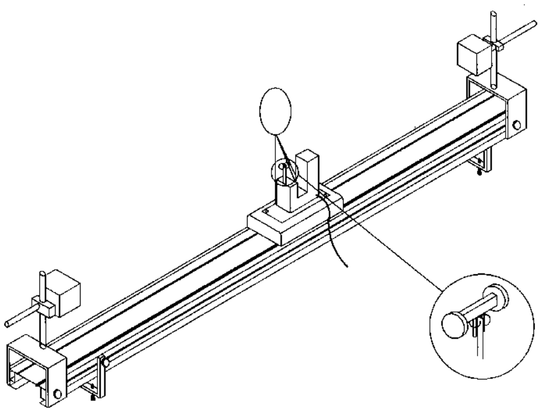


Figure 4: A dynamics cart is a deep square well potential.

The basic arrangement is the same as for the case of the spring oscillator except that the springs are disconnected and springy blocks are fixed to the end posts with Velcro. We have used variously, a sponge with a thin cardboard square glued to its faces, a blackboard eraser and other soft materials of varying coefficients of restitution. For the dynamics cart, a double faced hammer-like plastic plunger (inset, Fig.4) was designed to fit into the hooks of the Student Force Sensor to generate a neat signal on impact with the end blocks on either side.

Square Well Potential

Fig. 5 displays the force vs. distance graph for the bouncing off at the edges of the track. The students interpret these graphs qualitatively, relating the sudden changes in velocity to the force experienced at the walls of the well. They have difficulty, however, constructing the corresponding potential energy curve using the velocity data because of noise in the data and a significant loss of energy. Instead of using the velocity data, in this case, the students evaluate the potential energy as the area under the force-distance graph. The corresponding potential energy diagram is given in Fig.6. By comparing this graph with the harmonic well curve and the difference in the motion of the object in the two cases the students gain an understanding of how the shape of the potential well is indicative of the operative forces.

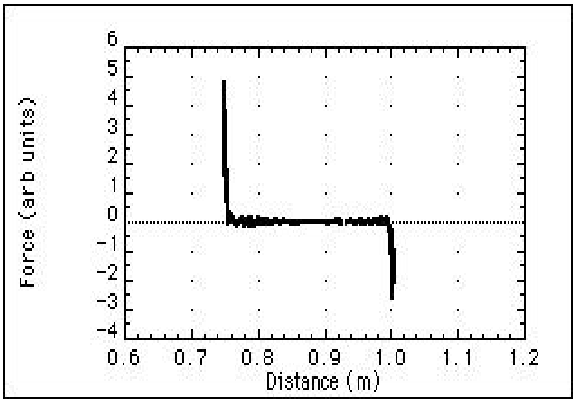


Figure 5: Force-distance graph for a cart bouncing off springy blocks at the ends of the track.

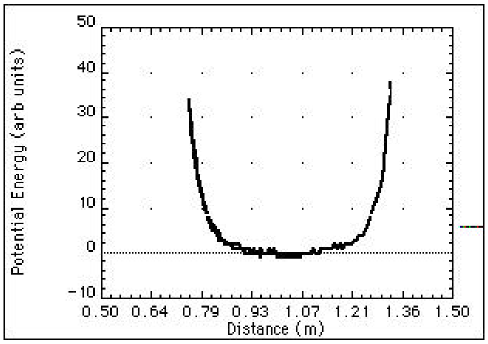


Figure 6: Square well potential diagram for the cart bouncing off springy blocks at the ends of the track.

2. Magnetic interactions

Some important potential shapes can be generated by the combined effect of magnets placed in attractive and repulsive modes along the track and interacting with an object moving along the track carrying a magnet. The interaction at a distance has an important advantage over the contact interactions which often have too much damping to allow the assumption of energy conservation even over a short duration.

In principle, while the measurement of position using the motion detector in real time allows a straightforward method of computing the velocity and hence the kinetic and potential energy functions for a nearly conservative system, in practice, even the slightest noise in the signal can lead to large jitters in the values of velocity and render this procedure useless. In a bid to circumvent this problem, we have used a Magnetic Field Sensor[10], which incorporates a Hall effect transducer, to generate directly the potential energy diagrams for one-dimensional motion. This sensor is mounted on the cart or the glider and as it moves through a configuration of magnets, the magnetic intensity is recorded as a function of distance. Since for a magnetic dipole placed in an external magnetic field of intensity B_x along the x direction, the net force experienced along the x direction is given by

$$F_x = -q_m l \frac{\partial B_x}{\partial x}$$

where $q_m l$ is the strength of the dipole, the potential energy

$$U(x) = -\int_0^x F_x dx = q_m l B_x$$

is simply proportional to the magnetic field intensity B_x. Thus, the measurement gives a direct on-line plot of the potential energy as a function of distance. The method offers a great deal of experimental simplification and permits an investigation of several potential shapes in a short duration class.

Dynamics cart system: Figure 7 gives a schematic diagram of the mechanical arrangement. We have used Neodymium magnets supplied by PASCO[11]. These magnets have a large magnetic moment and need to be used with due care and precaution. The magnets are held along or across the track at a fixed height using specially designed mounting platforms that slide into the edges of the track. The field magnet can slide along a slot in the platform and be fixed with any orientation in a horizontal plane parallel to that of the track. These degrees of freedom allow, (i) changing the strength of the magnetic interaction by varying the distance of the field magnet from the edge of the track, and (ii) changing the nature of interaction by rotating the axis of the magnet relative to that of the rider magnet. The clamping screws prevent the magnets from flying out under the influence of strong interactions. A flat plate screwed on the cart has a holder slot for sliding in the Hall probe tube casing. The rider magnet is fixed on this plate with Velcro. Again, the plate carries a post for attaching the reflector disk.

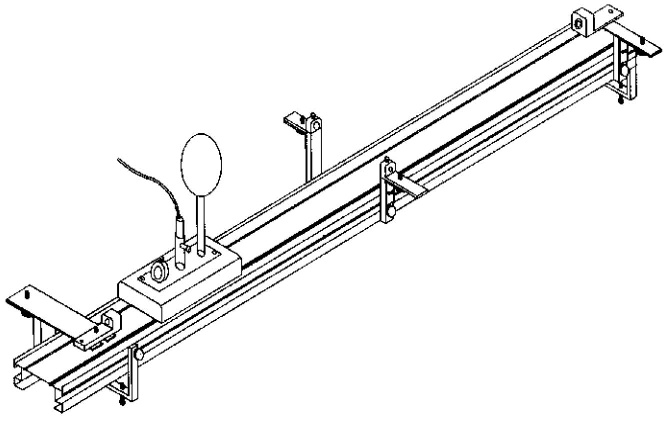


Figure 7: Mechanical setup for recording the interaction of a rider magnet on a dynamic cart moving through a configuration of field magnets arranged along a track.

Air track system: The basic arrangement using the air track remains unchanged from the one for the dynamics cart. In this case too, we have fabricated and used specially designed accessory pieces to hold the magnets along and across the track (Fig. 8). The hall probe is mounted on the glider using a simple holder that plugs into the central hole on the top plate of the glider. To decrease added mass we reduced the length of the acrylic tube holding the sensor to about 5 cm and also replace the thick output cable by thin flexible wires.

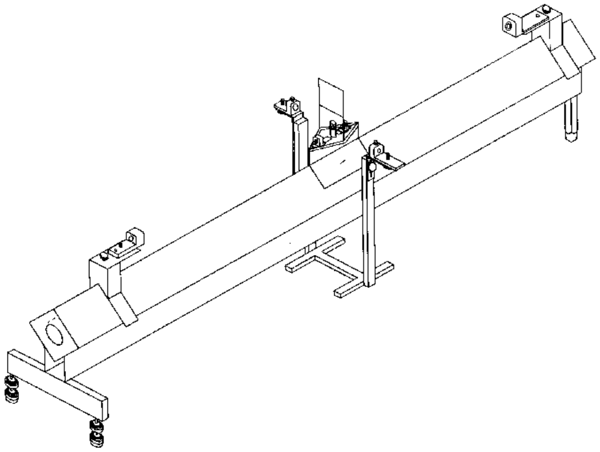


Figure 8: Mechanical setup for recording the interaction of a rider magnet on a glider moving through a configuration of field magnets arranged along an air track.

A good way of eliminating wobbling and unnecessary torque is by using symmetrical planar arrangements of magnets around the axis of motion and ensuring that the additional mass of the glider does not cause the center of mass to shift from the axis of the track.

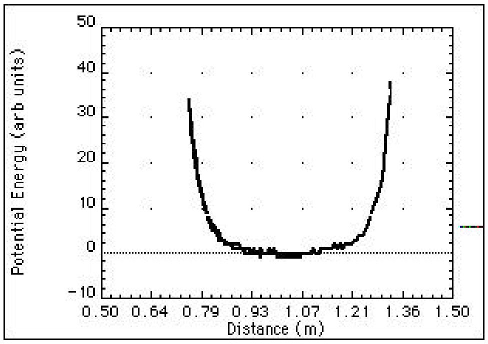
Potential well diagrams

The actual measurement of the magnetic intensity using the Hall probe is quite simple. Figure 9 displays a few illustrative configurations of the rider and field magnets and the corresponding potential energy diagrams generated through actual experiments. In these graphs, we have defined the potential energy in regions far from the positions of the magnets as zero and accordingly used the corresponding constant Hall probe reading to offset the measured values.

For each of these potentials, students obtain an interesting result by starting the cart moving with different velocities. Then, they see variations in the change in motion, regions where the cart is not able to travel, and the regions where it gets trapped and executes local oscillations. The students may also slide the cart gently across the region of interaction and retrace the potential energy curve. Relating the speed that must be imparted to the cart to cross a particular barrier or potential shape gives a kinesthetic experience of how an object moves in a potential well.

III. The Pedagogic Approach

The pedagogic approach we have followed involves determining, from experimental data, the potential energy of an object subject to a given force or set of forces. . The students use a computer data acquisition system to record the motion of the object using a motion detector in real time. For elastic interactions, they evaluate the kinetic and potential energy at each point along the path of the object, look at how the force varies as a function of position, plot the potential energy diagrams and interpret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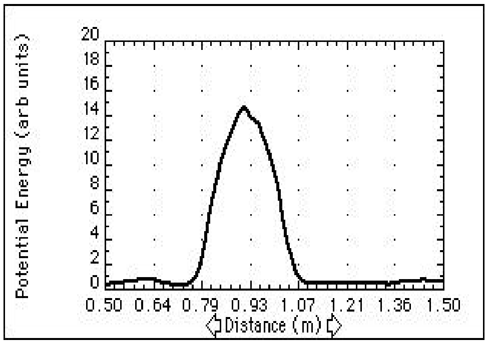
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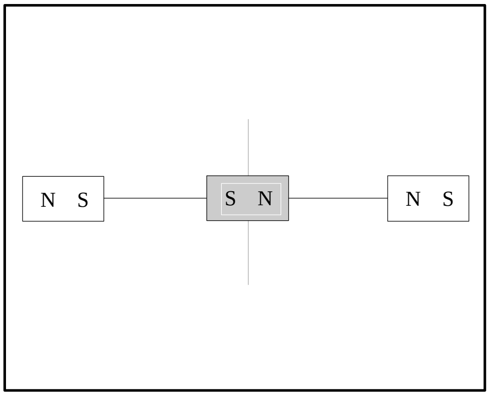
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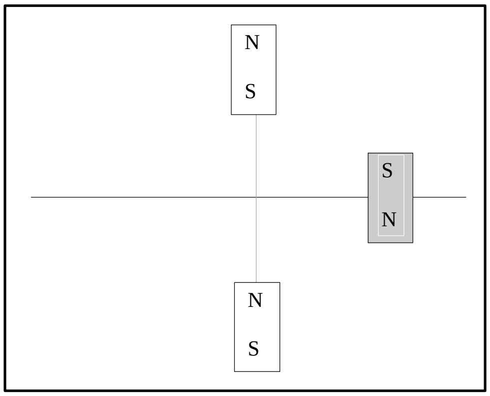
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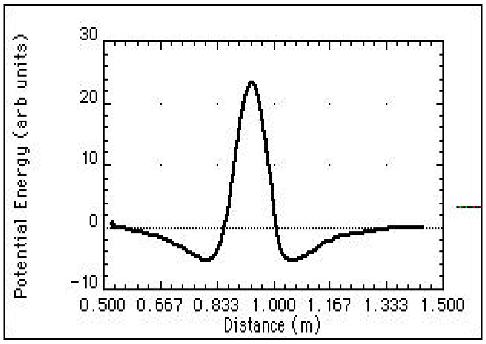
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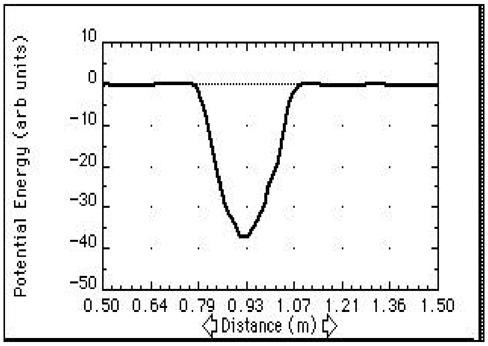
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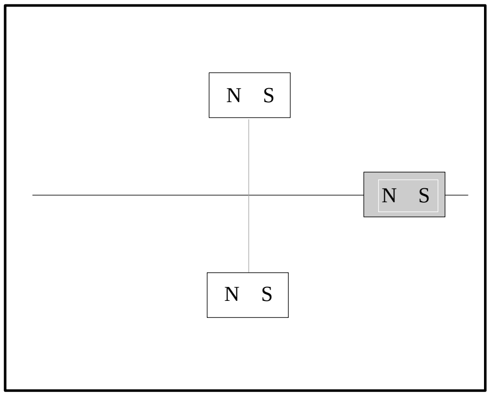
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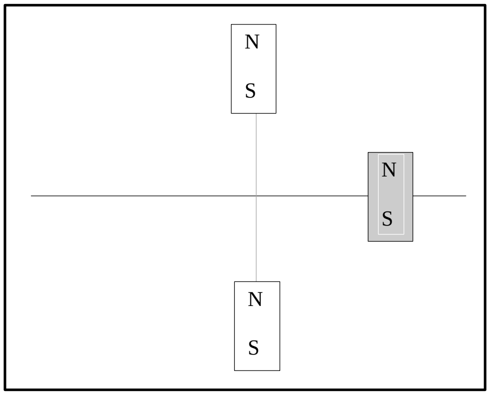
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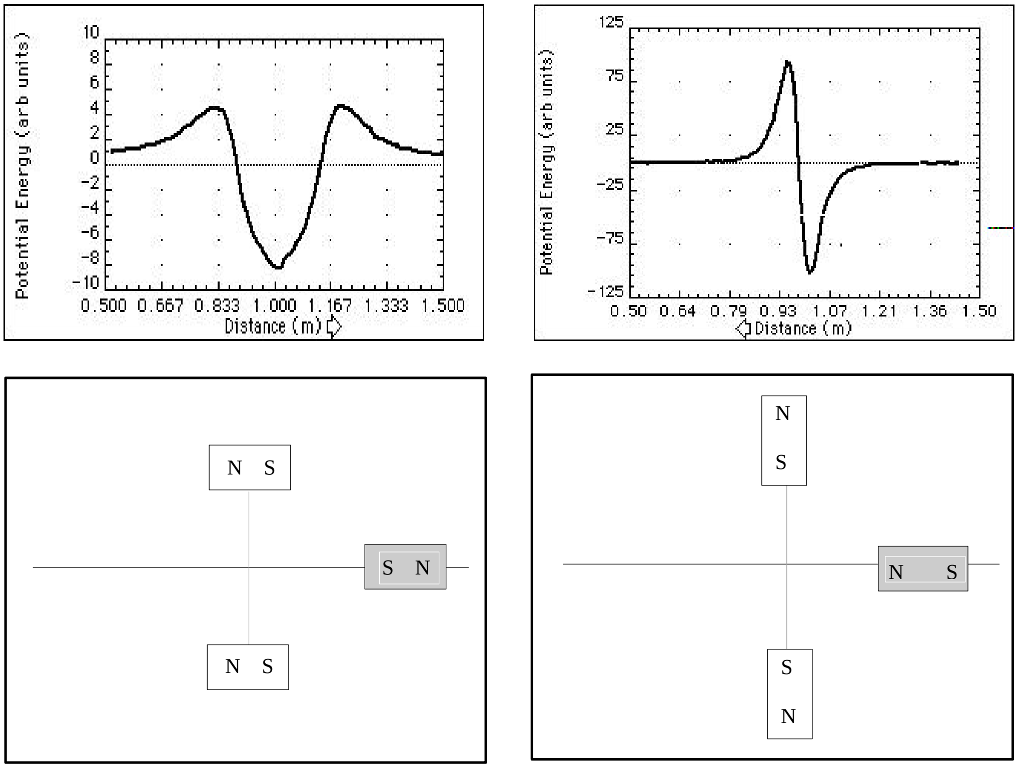
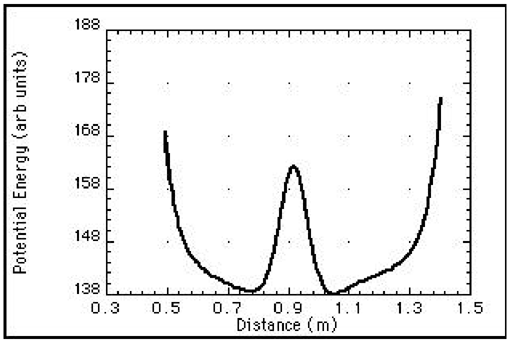
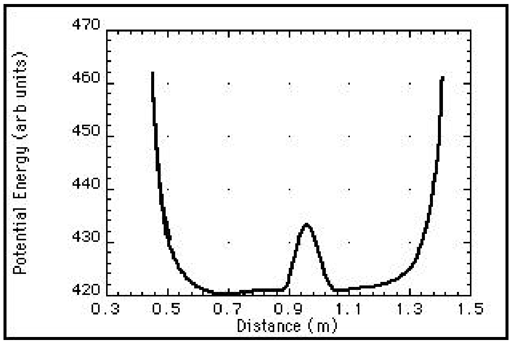


Figure 9: Potential Energy Diagrams. Direct measurement using Magnetic Field Sensor as a rider magnet (gray) moves on a track interacting with a configuration of field magnets.

the shape of the energy well to predict the motion of the object under varying initial conditions. Then, they construct a variety of potential shapes using a configuration of magnets. Using the Hall probe, they generate direct on-line plots of the potential energy diagrams. Using the potential diagram as a model of some unknown interaction, they explore how an object will behave in this potential. On the basis of the given picture, they try to predict the motion under different initial conditions and proceed to empirically verify their hypothesis and expectations. The arrangement allows posing open-ended problems wherein the students are asked to configure the magnets so as to create a potential in which an object can be trapped in a certain region or have some other specified motion. Pondering over motion in potential energy diagrams such as those in Fig.10 provides a casual introduction to idealized models of particle interactions the students are bound to encounter in quantum physics.





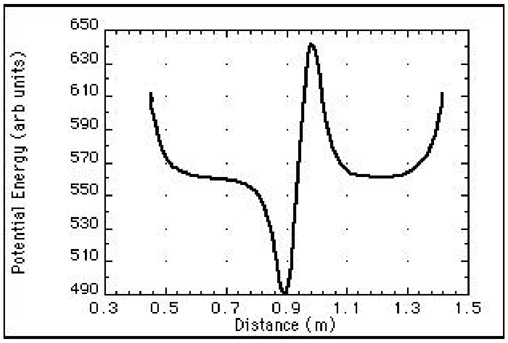


Figure 10: Students are asked to create magnet arrangements to create potentials such as these wells.

IV. Conclusions

These experiments and the accompanying material have been field tested in the classroom. The students response and feedback from classroom usage has been largely positive. The real time capture of data and immediate processing makes it easy to relate how different physical quantities change as a function of displacement. The sequence of experiments builds the bridge from tangible forces exerted by springs and springy blocks to more complicated force fields exerted by a configuration of magnets. It systematically leads the students from a concrete experience of forces in real physical systems to the abstract representation of their effect in potential energy diagrams. It allows them to move from specific experiments to general conclusions. We hope that this pedagogic route will prepare the students to extend the potential energy diagram concept to models of interactions in quantum physics.

The experiments described in this paper can be used at various instructional levels either as laboratory activities added to the traditional mechanics course or as a preamble to a quantum mechanics course. The emphasis can be either on the data capture and rigorous analysis to obtain potential energy functions from the force of interaction, or it can be on generating on-line pictures of potential energy diagrams and qualitative interpretation of motion of an interacting object.

A deliberate attempt has been made to use devices that are readily available from commercial suppliers. For a robust classroom arrangement, however, we found it convenient to fabricate additional accessory pieces. We have included a functional description of these in the paper. Additional information, pictures and the blue prints for machine fabrication are located at our web site at http://bluegiant.phys.ksu.edu.

V. Acknowledgments

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[5] S. G. Eckstein, “Verification of fundamental principles of mechanics in the computerized student laboratory,” Am. J. Phys.**58**, 909-915 (1990).

[6] Universal Lab Interface; Program Motion Version 4.5, Motion detector (U-MD); Student Force Sensor (Code SFS-DIN), Vernier Software.

[7] Dynamics System (ME-9249A), Air track (SF-9214) and assorted accessories, PASCO Scientific.

[8] Strain Gage Force Sensor Kit (SGK-DIN), Vernier Software.

[9] The total time of observation here is 10s. The extent of spreading due to frictional losses and the goodness of this fit is indicated by the correlation factor which is 0.97 for this data.

[10] Magnetic field sensor (Code MG-DIN), Vernier Software.

[11] Neodymium Magnets (EM-8621) PASCO scientific.