

Introducing Physics Majors to Computational Tools

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A course to introduce physics majors to several computational tools has been created. The material presented in this course provides the students with computational skills which can be used throughout their undergraduate careers. Our initial experience in teaching the course indicates that the course is generally successful at meeting its goals and shows some areas where improvements are required.

As we increase the use of computation for the physics majors at Kansas State University, we have several goals

- to maximize the students abilities to use computational tools,
- to minimize the amount of instructional time which must be used in regular physics courses for teaching computational skills, and
- to assure the students have computational tools available to them for a large part of their undergraduate careers.

With these goals in mind we have created a course in which the undergraduate physics majors are introduced to a variety of computational tools. While enrolled in this course the students learn to use a variety of tools which will be available to them during all future physics courses. By offering the course at the freshman and sophomore level we are seeking to reach the goal of making the tools available for a maximum period of time. At present, the course, Computational Tools for Physics, is still in the experimental and exploratory stage. My description here is based on the first time that it was offered, during the Spring semester, 1990.

The basic purpose of the course is to teach the students skills which can be applied to their study of physics. In our course we have chosen to focus on skills related to the use of several software packages rather than on programming skills. The selection of software for the course was made to span a broad spectrum of types of materials that the student might find available from commercial sources. The software needs to be able to address a variety of different types of physics problems which students will

face during their undergraduate careers. When they have finished the course, the students should be able to use software to do symbolic manipulations in algebra or calculus, numerical analysis, prepare graphs, use graphs to obtain insights to physical phenomena, and prepare reports and papers.

A major factor in the decision to exclude programming is the availability of other means by which students can become proficient in programming languages. Many of our physics students come to the university with some proficiency in either BASIC or Pascal. Students who wish to learn programming or to become more proficient than they already are can take a variety of courses from either the Department of Computer and Information Sciences or one of the departments in the College of Engineering. Thus, it seems appropriate for us to rely on these other sources for formal education in computer languages and to concentrate on the use of software packages which can be applied to physics learning and physics research.

Our avoidance of computer programming is also motivated by a general attitude which seems to prevail among our students. They frequently seem to believe that the only way to solve a problem with a computer is to start writing code. Because the process of defining a problem, writing code, and debugging the code can be a tedious and frustrating job which consumes the student for weeks at a time, we feel that students should be introduced to other ways to accomplish similar tasks. We feel that they will have ample opportunity to write programming code and learn programming languages in other endeavors, so we have completely avoided it in this course.

I. SOFTWARE TOOLS

The symbolic algebra and calculus package was Derive[1]. This program is a rather powerful symbolic manipulator which has a friendly user interface. In selecting Derive we concluded that students could be introduced to it and begin using it productively much more rapidly than some of the more extensive symbolic manipulators which seemed to have a longer learning curve. We were also impressed by Derive's rather low purchase price (\$200 for a single copy or \$75 per station for a network). This price was important for our budget but also was low enough that we could expect some of our students to purchase a copy for their own systems.

Some numerical work was completed using a spreadsheet program. Quattro, Student Edition[2], was the recommended spreadsheet although students were allowed to use any other spreadsheet with which they had some familiarity. The spreadsheet program was not installed on our network. Instead, each student was required to provide his or her own copy. Quattro was selected after a comparison of the student editions of Quattro, Excel[3], and Lotus 1-2-3[4]. Functionally all three programs seemed to be identical for our purposes. Quattro's price (\$39.95) was approximately \$10 less than its nearest competitor. Thus the decision to use Quattro over other similar spreadsheets was made on a financial basis.

MATLAB[5] was chosen for general graphical and numerical work. Several other packages such as Gauss[6] and Speakeasy[7] were also considered. Each seemed to have some advantages. In particular the Physics Department VAXs have Speakeasy on them. Thus, the selection of Speakeasy would provide easy transition to some research efforts. However, we share our facilities with the Mathematics Department and our budget was somewhat limited. Thus, while the physics faculty would have preferred Speakeasy because of previous familiarity, a compromise with the Mathematics faculty resulted in the selection of MATLAB.

In addition to these three programs for addressing physics problems we selected a scientific word processing program so that students could write reports, including equations in a reasonably straightforward manner. Our choice for this activity was ChiWriter[8]. While both the physics and math departments use \TeX extensively on their VAX and MS-DOS systems, we felt that the learning curve for high productivity in \TeX was longer than we wished. Our goal was to have a program which students could use very easily and still be able to produce complicated

equations in a readable manner. Thus, we selected ChiWriter which is relatively easy to use but does not produce quite the quality of \TeX .

To help students learn the MS-DOS system we made available a shareware program which would provide help screens for all of the common MS-DOS commands. We used this in the spirit in which shareware is offered by making it available and seeing if students would use it. We found that they did not and have removed it from our system.

These sets of software provided the tools for the course.

II. THE CLASSROOM ENVIRONMENT

Math-Physics Computer Classroom and Laboratory was created with funds provided by the state of Kansas. The classroom provides an environment in which instruction may occur while students are seated at a computer. In this environment a recitation-laboratory atmosphere may be established. A teacher may provide some instruction and immediately ask the students to try it out. While the students are working, the instructor can walk around the classroom and find out what specific problems occur when the students are doing it for themselves. Likewise, one can establish an exploration type atmosphere by presenting a new piece of software or a new aspect in one of the packages and then say, "see what you can do with that". In this manner immediate feedback occurs and we do not find ourselves in a position where the students must go to a separate computing facility in order to discover that they missed an important point during the class.

The hardware for the system includes 15 advanced MS-DOS machines and a file server. The MS-DOS machines are based on an 80386 CPU with four megabytes of RAM and "super" VGA graphics. All of the computers are equipped with a 1.2 megabyte 5 1/4 inch floppy drive. Five of the machines also have a 1.4 megabyte 3 1/2 inch disk drive. Fourteen of the machines are arranged in rows all facing toward the front of the room and are used as student workstations. The fifteenth machine which has a 40 megabyte hard disk in addition to the other equipment is the instructor's station and is on a mobile cart at the front of the room. An overhead projection system is also attached to the instructor's computer.

A Sun SPARCstation is utilized as the file server for the system. Each of the 386 machines is connected via PC-NFS to the SPARCstation. The SPARCstation is also connected to a Hewlett-Packard Laser-

writer II and a dot matrix printer as well as the university Ethernet. In addition to serving as a file server the SPARCstation can also be used to teach students how more advanced workstations can be used in physics and mathematics learning and research. Once students reach some of the limitations of the MS-DOS machines and accompanying software, they can be shown how other materials can be effective on the UNIX-based Suns. We do not have sufficient funds to provide a classroom with Sun systems. However, students can learn to use some programs in the laboratory and then apply them to their physics either by using the Sun in the classroom when no classes are using it or by working in the university's public access Sun laboratory.

This physical environment provides a pleasant atmosphere in which to teach and learn. In introducing students to the various software packages the ability to have the students work immediately on new ideas became critical. Certainly, hours of student and instructor time was saved by being able to diagnose the student's lack of understanding quickly.

III. COURSE OPERATION

The course began with an introduction to algebra on the computer. The choice of beginning with algebraic manipulations was not made for any deep pedagogical reason. In fact, I had intended to begin the course by looking at spreadsheet applications in physics. However, an error on my part resulted in the bookstore ordering copies of the student edition of Quattro late, and thus it was not available at the beginning of the semester. In retrospect this choice of software was a good one even if it was not for the right reasons.

The approach to teaching the use of Derive (and all of the packages) was to provide a minimal introduction so the students could get started. Then, the use of further and more sophisticated applications and functions was motivated by physics problems which could be solved. For Derive the bare bones of solving algebraic problems and performing integrals and differentiation were presented to the students. Then they were given problems from an introductory calculus-based physics course to solve with the package. To provide students with problems I selected worked examples from books such as Halliday and Resnick[9], Ohanian[10] and Eisberg and Lerner[11]. By providing worked examples and asking students to set them up in Derive I felt that I was avoiding some of the problems which would occur if, during class, one of the students could not work the example. For

the most part this was a successful approach. It motivated the students to learn the various functions which were available within Derive and to become somewhat proficient with them. However, it did lead to the comment that Derive seems to be useful only if one already knows the answer to the problem. Thus, next year I will use a larger number of problems for which "we do not know the answer".

The next section of the course introduced the students to the use of spreadsheets in physics. Primarily we were interested in looking at methods for doing integrals numerically and solving differential equations numerically. It may seem somewhat ludicrous to use a student version of a spreadsheet for these operations when one has a sophisticated program such as MATLAB "waiting in the wings". However, learning how numerical integration algorithms worked and difficulties which might occur was an important objective in this study. The spreadsheet provided an excellent opportunity for students to learn the basic concepts and to explore methods of numerical integration.

In developing materials for the spreadsheet portion of the course I relied heavily on two rather old books by Eisberg[12] and Eisberg and Hyde[13]. In these small paperbacks Eisberg presented numerical methods and a series of exercises and explorations for pocket calculators. While the hardware for which Eisberg wrote his book is somewhat old, the pedagogical techniques and information is extremely useful.

The students were finally introduced to MATLAB and its various capabilities. This mathematics and graphical package has been described by Cook[14] and others. In introducing MATLAB to the students we relied heavily on handout materials which have been used at Lawrence University and were kindly provided to us by the physics faculty at Lawrence. In addition, the students used MATLAB to address the problems which we had already tackled with spreadsheets and adaptations of problems from books such as Thompson[15] and Gould and Tobochnik[16]. Thus, students could compare their results using a spreadsheet on a numerical integration with the results of MATLAB and discuss how the two approaches were different and why one package might work better than another.

The introduction and exploration of the three major software packages required slightly less than two-thirds of the semester. This time period was somewhat longer than I would have liked and can be attributed in part to my own lack of experience with some of the software and with the network. While

I would not wish to “save time” by decreasing the time during which students explored the packages, I suspect that I can diagnose misconceptions and difficulties somewhat more quickly by developing better teaching strategies. These pedagogical approaches will still allow students to make errors and learn from those errors. However I anticipate that I will be able to arrange it so that these activities require a shorter amount of time. In the future I anticipate using about 50% of the course to introduce software tools.

As the students were introduced to the various software tools, they began with a software package and were presented with physics problems which could be approached with that package. Of course, in real physics research or learning we do quite the opposite. We have a problem, and we must decide what tool is most appropriate for that problem. Thus, in the last one-third of the course we selected problems and discussed how to attack them. Then, the students would go forward with their various tools and see what they could learn. In some ways this part of the course was most enjoyable because both the students and the instructor were able to suggest problems to undertake. Further, the students and the instructor (particularly the instructor) were able to find problems which showed us the limitations of all of the software packages. The net result was that we had an enjoyable time discussing problems in areas ranging from mechanics to quantum mechanics and learning how to recognize non-sensical solutions if they appeared.

The final two weeks of the course were devoted to types of problems which required combining two or more of the software packages in order to reach a successful resolution of the problem. Most often, we would set up a problem and reach an algebraic solution by using Derive and then need to complete a numerical analysis with MATLAB. For example, we attempted to model mathematically the pushing of a child on a swing by using several different types of pulses for the periodic driving force. Some of these forces had very rapidly changing magnitudes and were not easy to work with in the differential equation solvers of MATLAB. Therefore, students would perform a Fourier analysis of the driving force and use the Fourier components in solving the differential equation via MATLAB. Pedagogically this was a very useful set of exercises because it helped the students understand some of the limitations of the programs and of the various techniques for solving them. At the same time it could provide a useful motivation for moving to other materials which are available on our Sun workstation.

Table 1: Weighting of various components of final grade.

Two tests	30%
Final exam	20%
Project	25%
Homework	25%

IV. TESTS, GRADES AND PROJECTS

Student grades were based on homework, tests and a project. How each of these components contributed to the final grade is presented in Table 1.

Homework was assigned to be due approximately every ten days during the course of the semester. Students were expected to complete the homework assignments outside of class and turn them in on the due date. To accomplish this, the student needed to have access to the classroom during hours when no classes were meeting in it. Students submitted homework by leaving magnetic copies on the system hard disk. The instructor could then grade directly from those copies and transmit his comments electronically to the student. For the most part this procedure proved to be an efficient means to collect and grade homework for the class.

The examinations in the course proved to have a rather unusual format. An examination would begin at the beginning of one of the regularly scheduled class hours. Students could work on the exam during that class period and during the class period immediately following the one in which the exam began. At the end of the second class period the examination must be submitted for grade. Between the two classes the students were allowed to work on the exam if they so desired.

This method was devised because it became clear that a test which provided a reasonable assessment of the student’s ability could not be completed during a single class period. By administering the exam during a regular class I provided the students an opportunity to ask questions about the test items and to see if there were any general problems or misconceptions about the interpretations of the examination. Because students could work on the exam outside of class, the second class period would probably not have been necessary. However, it did assure me that every student had at least two hours access to the

Table 2: Software Chosen for the Final Exam.

	Problem 1	Problem 2	Problem 3	Problem 4
Derive alone	3	–	1	2
MATLAB alone	0	–	0	3
Derive and MATLAB	3	–	4	1
Spreadsheet alone	0	–	0	0
Derive, MATLAB and spreadsheet	0	–	1	0

machinery for the purpose of completing the exam.

For each test the students concentrated on one or two of the software packages. On the final examination no suggestions for software packages were given. The students were free to choose any package to address each problem. The examination is included as an appendix. The software of choice for each problem is given in Table 2. (Students were required to complete three of four problems; all students omitted problem 2.)

For the term project each student was to select a problem of interest to him or her and then use one or more of the software packages to solve it. The problem was to be of sufficient difficulty that it required more than a simple homework solution. The original concept was that shortly after the students had completed an introduction to all of the software packages they would submit a short description of their proposed project. That description would be reviewed by the course instructor and a critique returned. The students would then revise the description and submit a somewhat longer description of the problem and the proposed solution. This second version was to be distributed to all students in the class so that “peer reviews” would be made available to each student. Then, the student would actually use the software package or packages to complete his or her project. In this way we would approach the projects in a manner that approximated doing research in physics.

Unfortunately, the system needs serious refinement before it will work successfully. Part of the failure was the length of time needed to introduce all of the software packages. With approximately one-third of the semester left we had insufficient time to provide the various drafts of the proposed project and to actually tackle the project in a meaningful way. Further, the first draft descriptions provided by the students were sufficiently vague that the instructor found it

Table 3: Summary of Students’ Projects.

<i>Topic</i>	<i>Software</i>
Moments of Inertia of object with unusual configurations	Spreadsheet
Radioactive Decay	Derive and MATLAB
Projectile Motion with air resistance and variation of acceleration of gravity	MATLAB
Schrödinger’s Equation for a rigid rotor	Derive
Rutherford Scattering	Derive
Problems in Thermal Physics and Planck’s Radiation Law	Derive
Square Well Potential with a Coloumb Force	MATLAB

difficult to provide useful feedback. Next year, more guidance will be provided on what is an appropriate first draft, and an improved schedule will offer more time for the feedback from the students' peers. In practice this year the peer review was eliminated due to lack of time. The students moved directly from their first draft proposal and comments from the instructor to the final stages of the project. A list of student projects is presented in Table 3.

V. GENERAL COMMENTS AND CONCLUSIONS

The most important issue raised with this type of course is whether these tools should be introduced to physics majors in a separate course or as an integral part of other courses which the students take. At the beginning of this paper I argued that a separate course was appropriate because it decreased the time taken away from instruction on the main objectives of the other physics courses. Some of my colleagues have argued that the formal instruction in computation is not needed and that students should simply become proficient with computational tools in the same manner that they learn proficiency with a pocket calculator. However, my experience indicates that students cannot obtain proficiency with these tools unless some type of formal instruction is provided. Thus, a course of this nature or some formal instruction within the regular physics courses does seem to be necessary.

The difficulty with teaching the materials as a separate course is the lack of a context for the materials. While the students recognize the physics of the problems which are presented and realize that we were tackling problems which generally were not approached in many undergraduate courses, each of the problems was an isolated example. Thus, the software rather than the physics became the focus of the course. I see no reason that this should be a difficulty, but I confess to being somewhat uncomfortable with software as the primary focus of the course.

Generally, the computational tools which we selected were appropriate for the course and did provide the range of abilities to enable students to tackle a broad spectrum of problems. The same set of software tools will be used again during the next offering of the course.

Overall, I felt that the course did successfully introduce these students to a set of useful computational tools which they can apply to their learning of physics. I anticipate that all of the students who completed the course will continue to use these tools

for the remainder of their undergraduate education. The critical test of the usefulness of the course at our university will come when other faculty realize that the students now have this capability. If the faculty broaden the coverage in the other courses to take advantage of the students' abilities with computational tools, this course will be a success and will become a regular part of our undergraduate curriculum. Because change of this nature does not happen overnight, we will only learn the full impact of this course by watching the evolution of our undergraduate physics major over the next few years.

APPENDICES

These appendices record the final examination given in the described course and also summarize the hardware and software available to physics majors at Kansas State University.

A. FINAL EXAM

ALL PROBLEMS HAVE EQUAL WEIGHT. COMPLETE ANY THREE OF THE FOUR PROBLEMS. If you get ambitious and do all four, your best three will be included in your grade.

1. The James R. Macdonald Laboratory can create ions with most of the electrons stripped off. For example, an argon ion with only one electron is a possibility. Consider such ions which have only one electron orbiting a nucleus of arbitrary Z . To a first approximation we can treat the electron-nucleus system as a hydrogen atom with $Z \neq 1$. However, this approximation will decrease in value if the electron has an appreciable probability of being inside the nucleus.

(a) Investigate the probability of the electron in the ground state of a one-electron ion being inside the nucleus. Obtain an algebraic expression for the probability. Use the following assumptions:

The wave function of the electron is given by the hydrogen wave function suitably corrected for increasing Z .

The radius of the nucleus is given by $r = RA^{1/3}$, where A is the number of nucleons in the nucleus and R is 1.2×10^{-15} meters.

(b) Obtain numerical results for one-electron atoms with the following nuclei:

$${}^1_1\text{H}; {}^2_1\text{H}; {}^4_2\text{He}; {}^{12}_6\text{C}; {}^{34}_{18}\text{Ar}; {}^{109}_{47}\text{Ag}; {}^{238}_{92}\text{U} \quad (1)$$

2. The Exploratorium, a science center in San Francisco, has an interesting activity to demonstrate resonance. A very massive piece of iron and concrete is suspended from the ceiling. Visitors to the Exploratorium stand behind a fence and pull on a string which is attached to the large mass by a very small magnet. Thus, they can pull with only a small force or the string detaches from the large mass.

(a) Suppose that the large mass is not moving initially, and one visitor pulls gently on a string. Using the data below and a small angle approximation, provide a graph and a table of the displacement and velocity as a function of time.

- Mass: 200 kg
- Length of suspension: 10 m
- Air resistance constant, b : 10 N/(m/s)
- Pulling force $F(t)$:
 - 1 N (pendulum moving toward visitor)
 - 0 N (pendulum moving away)

(b) Repeat the calculations for a situation where the angle of oscillation becomes large.

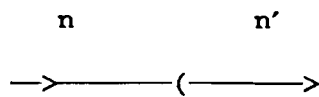
3. One can use matrices to treat optical systems in a manner similar to the treatment of ray tracing. However, the matrix method does not require that the lenses be thin. Here, I will discuss how to use the method for situations where the light rays are near the axis of the lenses and angles are small (i.e. $\sin a \approx \tan a \approx a$.)

When light moves from one medium to another, we define the refraction matrix as

$$R = \begin{pmatrix} 1 & 0 \\ \left(\frac{n_1}{n_2} - 1\right) \frac{1}{r_1} & \frac{n_1}{n_2} \end{pmatrix}$$

where n_1 is the index of refraction of medium which the light is leaving, n_2 is the index of refraction of the medium which the light is entering and r_1 is the radius of curvature of the surface

For example



$n = n_1$, $n' = n_2$ and r is the curvature of the surface. When light travels within one medium, we define the transmission matrix as:

$$T = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$$

where d is the distance over which the light moves.

The system matrix, S , is the matrix product of all of the refraction and transmission matrices. So, the system matrix for a single thick lens would be

$$R_2 T_{12} R_1 = S \tag{2}$$

Note the order of the matrices reads from right to left. The refraction matrix for the first surface is on the right, followed to the left by the transmission from the first to second surface, and the refraction for the second surface.

Once the system matrix is determined, we can find the location, i , of the image by the equation

$$i = -\frac{ao + b}{co + d} \tag{3}$$

where o is the distance from the center of the lens to object and

$$S = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

An inexpensive camera will use a pair of lenses to decrease chromatic aberrations. These lenses will have the following properties.

	Convex Lens	Plano-concave
Radius of curvature	20 m	-20 cm, ∞
Index of refraction	1.5	1.63
Thickness on axis	x	0.4 cm

Use the system matrix method to determine the image location as function of the thickness, x , of the converging lens. Allow this lens to vary from a thickness of 0.2 cm to 20.0 cm. Present data as both a table and a graph.

4. Simple radio transmitting antennae are half-wave dipole antennae. The average energy flux radiated from such an antenna is

$$\langle S \rangle = \frac{\langle I^2 \rangle}{4\pi^2 \epsilon_0 c r^2} \frac{\cos^2(\frac{\pi}{2} \cos \theta)}{\sin^2 \theta} \tag{4}$$

where $\langle I^2 \rangle$ is the root-mean-squared current delivered to the antenna and other variables have the standard meanings in electricity and magnetism. The average radiated power for the antenna is

$$\langle P \rangle = \int_0^\pi \langle S \rangle 2\pi r^2 \sin \theta d\theta \tag{5}$$

(a) Plot the average energy flux in two ways: First, present a plot of the angular distribution of $\langle S \rangle$; then, construct a three dimensional plot showing the variation in $\langle S \rangle$ in x - y plane. A reasonable value for $\langle I^2 \rangle$ is 50 amperes².

(b) In analogy with other alternating current expressions the equation for $\langle P \rangle$ is frequently written as

$$\langle P \rangle = \langle I^2 \rangle R \quad (6)$$

Evaluate R which is called the radiation resistance.

H. HARDWARE

The Departments of Physics and Mathematics share a classroom which contains 15 MS-DOS 80386 machines with 4-MB RAM, co-processors, and VGA graphics. These computers are connected through a local area network (PC-NFS) which is served by a Sun SPARCstation. The SPARCstation is also available to students and is connected to the campus Ethernet. The facility which houses these computers is located in a classroom environment so that classes may be taught while students are using the computers. Both dot matrix and H-P Laser Writer printers are available.

The Physics Department also maintains a VAX 750 which students and faculty can use via terminals and modems. All faculty have terminals, microcomputers, and/or workstations in their offices. Additional computing facilities ranging from PCs to VAXes are available in research laboratories and Departmental offices.

The campus maintains an IBM mainframe operating under CMS. A large number of public terminals are located throughout the campus. These terminals provide access to the Physics Department VAX, the mainframe and a number of other computers.

Public microcomputing facilities, which are available to all students, include about 100 MS-DOS and a few Macintoshes. A public UNIX system which includes a Solbourne server and four Sun SPARCstations is available to students.

Public facilities are open 24 hours per day, seven days a week.

S. SOFTWARE

See compilation on several pages at the very end of these proceedings.

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