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INVESTIGATING THE EFFECT OF THE MAGNETIC FIELD FROM A WIRE ON A COMPASS

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Abstract

This apparatus has two primary applications. The first is in teaching the concepts of resonance in the context of magnetism, which can be applied to the teaching and understanding of magnetic resonance imaging. The second investigates the direction of the field around a current carrying wire and the superposition of magnetic fields. The compass deflection angle can be measured as a function of the distance from the wire and compared to theory.

Construction of Apparatus:

This apparatus is constructed from a 30 cm. square piece of Plexiglas attached to short legs. One of the desired features of this apparatus was to be able to see a compass below the wire as well as one above. Therefore, Plexiglas was chosen because it is clear. However, we have also made this apparatus out of foam core and found it functioned identically with the disadvantage of not being able to see through it. (The foam core version is very cheap and easy to make.)

A groove in the shape of a right triangle has been cut in the Plexiglas so that a wire can be placed below the surface, allowing a compass placed above the wire to sit flat. In our apparatus, the side of the triangle is approximately 25 cm. long. At one corner of the triangle, a switch is placed. One of the other corners has two terminals for attaching the battery. The wire is placed in the groove and attached to the switch and battery such that when the switch is closed the circuit is complete. The wire can be held in the groove with either clear glue or small piece of tape.

We investigated the need for a resistor in the circuit and found that any resistance in addition to the internal resistance of a 6 V battery adversely affected the function of the apparatus. With a 6 V battery and only its internal resistance, the current is around 2.5 A giving a magnetic field near the wire comparable to the Earth's.



Use of Apparatus:

Many physics phenomena can be investigated with this simple apparatus. We have used it for two primary purposes:

1. To introduce resonance phenomena in the context of magnetism as a means of investigating the physics principles related to Magnetic Resonance Imaging.
2. To investigate properties of the magnetic field around a current carrying wire and the superposition of this field with the Earth's magnetic field, including the strength of the total magnetic field as a function of distance from the wire.

Resonant Phenomena

This apparatus was initially developed to help teach some of the physics that forms the foundations of nuclear magnetic resonance (NMR) and, thus, magnetic resonance imaging (MRI) [1, 2]. In these phenomena, the nuclei in a person's body resonate at a frequency dependent on the type of atom and the strength of an applied magnetic field. In addition, there is an electromagnetic wave which "kicks" the nuclei at their resonant frequency. These three things (nuclei, field, and electromagnetic wave) are analogous to the three parts of this apparatus (compass, field, and the field due to tapping the switch). More specifically, as shown in Figure 1, the resonant frequency of a non-damped compass placed above a wire is due to the type of compass and the strength of an external field due to either the Earth or a bar magnet placed on the apparatus. When the switch is tapped at the resonant frequency, the compass needle will spin. Thus, this apparatus can form the basis of a hands-on activity to teach MRI. (Such an activity is in development at Kansas State University as part of the Modern Miracle Medical Machines project.)

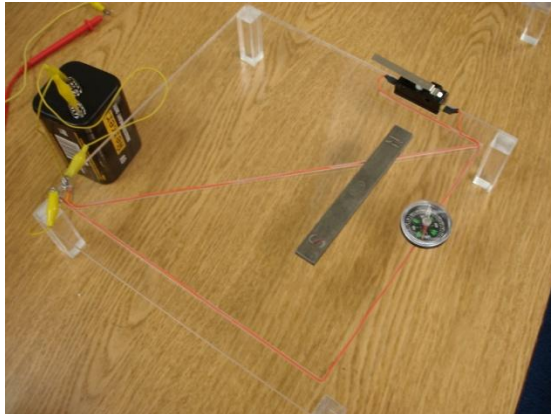


Figure 1 Picture showing the experimental set-up for changing the resonance frequency of a compass placed above the apparatus.

Mathematically, this can be seen by looking at the torque on a magnetic dipole, m , due to an external magnetic field (B_0), [3, pg 308]:

$$\vec{\tau} = I\vec{\alpha} = -\vec{m} \times \vec{B}_0$$

where I is the moment of inertia of the compass needle and α is its angular acceleration. (Note: B_0 is the field due to the Earth or a bar magnet placed on the apparatus. It is not the field due the wire.) This equation is analogous to the equation for the oscillation of a pendulum [4, pg 247]. By analogy, the oscillation frequency is thus:

$$f = \left(\frac{mB_0}{I} \right)^{\frac{1}{2}}$$

While it may seem feasible to use this formula to calculate the oscillation frequency, the magnetic dipole moment is, in general, an unknown. However, using the apparatus it is possible to measure the frequency (typically around a Hz) and then calculate the dipole moment.

As can be seen from the equation above, the oscillation frequency of the compass is dependent upon the strength of the external magnetic field responsible for the restoring force and the type of compass (both dipole moment and physical characteristics), but not on the field due to the wire. The field due to the wire gives the oscillation its initial amplitude (like pulling back on a pendulum) and can be used to kick the compass into resonance by tapping the switch at the compass' natural oscillation frequency. The dependence of the oscillation frequency on the external field can be investigated by adjusting the distance between a bar magnet placed on the apparatus and the compass, as shown in Figure 1.

Magnetic Fields and Superposition

One of the most obvious uses is the investigation of magnetic fields, their direction, and superposition. Because the magnetic field of the Earth will always influence the compass, the magnetic field direction indicated by the compass when the circuit is closed is the superposition of the field due to the Earth and the field due to the wire. Even so, a great deal about the magnetic field due to the wire can be determined by observing how compasses behave when the circuit is closed. For example, if the wire is oriented North/South such that the compass is parallel to the wire when the circuit is open, then the observed deflection of a compass below the wire is in the opposite direction of a compass above the wire, as shown by compasses A and B in Figure 2.

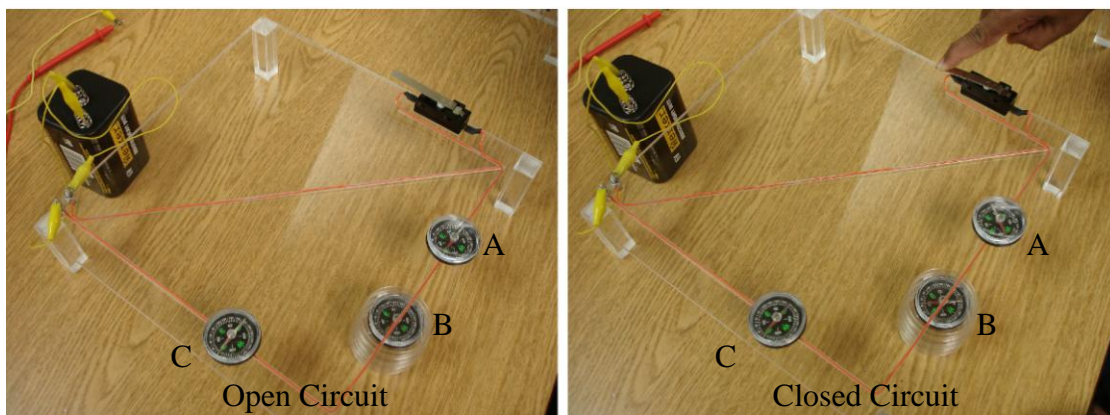


Figure 2 Three compasses placed above or below the apparatus when the circuit is open and closed. Because compass A is above the wire, it deflects in the opposite direction of compass B which is placed below the wire. Compass C does not deflect because the torque on it is zero.

Because the wire runs in a right triangular groove, if one side is oriented North/South, then the other side is East/West. A compass placed either above or below the East/West wire will not deflect when the circuit is closed, as shown by compass C in Figure 2. The knowledge that torque is zero when the force and the moment arm are parallel (or, in this case when the magnetic field due to the wire and the magnetic moment of the compass are parallel) and these two observations can lead to the conclusion that the magnetic field due to the wire is perpendicular to the wire and directed in opposite directions above and below.

The compass will deflect less as it is raised above the wire. If the current in the circuit is measured along with the compass deflection at various heights, then the measured compass deflection angle can be compared to a theoretically calculated value. However, because the magnetic field direction indicated by the compass deflection when the circuit is closed is a superposition of the Earth's magnetic field and the wire's, the deflection angle is not as simple as the $1/r$ dependence of the field due to the wire alone [4, pg 527]. The theoretical value is calculated using the geometry shown in Figure 3.

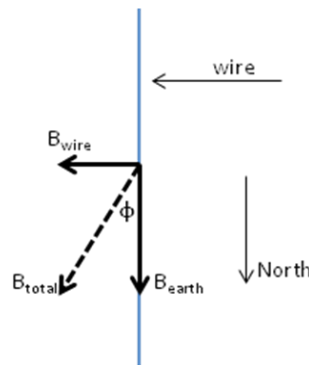


Figure 3 The geometry for calculating the dependence of the total magnetic field on the distance above the wire

(The Earth's magnetic field at many locations can be found online at <http://www.ngdc.noaa.gov/geomagmodels/IGRFWMM.jsp>. This website gives the horizontal and vertical components as well as the total field.) The deflection angle, ϕ , as a function of distance above the wire, r , for the total magnetic field is given by

$$\phi(r) = \tan^{-1} \left[\frac{B_{wire}(r)}{B_{earth}} \right] = \tan^{-1} \left[\frac{\mu_0 I}{2\pi r B_{earth}} \right]$$

where B_{wire} is the magnetic field due to the wire, B_{earth} is the horizontal component of the Earth's field, μ_0 is the permeability of free space, and I is the current in the wire. In this analysis, the assumption has been made that the field due to the wire at the compass is horizontal and therefore the total magnitude is used in the calculation. With the exception of the lowest point, good agreement between theory and data was achieved, as shown in Figure 4. The error bars on the measured value are ± 2.5 degrees and represent the precision of the compass readings. Our best guess on the disagreement for the first point is that the assumption about the direction of the magnetic field due to the wire has broken down.

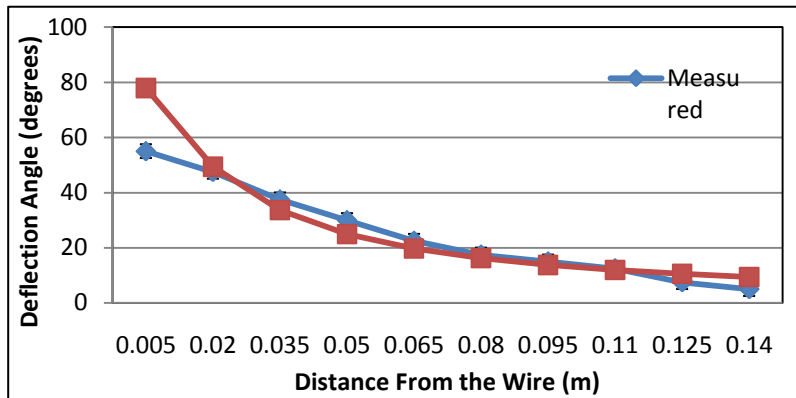


Figure 4 Measured and theoretical deflection angle (measured from North) for a compass above a current carrying wire as a function of the distance from the wire. The error bars on the measurement are ± 2.5 degrees, the precision of the compass readings.

There are two main applications for this apparatus. The first is the investigation of resonance in the context of magnetism. This application works well to introduce the physics concepts needed to understand magnetic resonance imaging. The second application utilizes the apparatus in the investigation of the properties of the magnetic field due to a wire and the superposition of two magnetic fields. The dependence of the total magnetic field on the distance from the wire can be measured and compared to theory.

1. Joseph Hornack. *The Basics of MRI*. (J. P. Hornack, 1996)
<http://www.cis.rit.edu/htbooks/mri/>
2. <http://www.simplyphysics.com/MAIN.HTM>
3. Roald K. Wangsness. *Electromagnetic Fields, 2nd Ed.* (John Wiley and Sons, 1986)
4. Jeff Sanny and William Moebis. *University Physics*. (WCB Publishers, 1996)

Parts List:

Item	Source	Part Number	Total Cost	Makes ³	Unit Cost ⁴
Acrylic Sheet	plastic-craft.com		\$66.58	5	\$13.32
Acrylic bar 0.75 x 0.75	plastic-craft.com		\$16.46	5	\$3.30
1 in. 6-32 screws (2 needed)	Ace Hardware		\$0.50 ¹	1	\$0.50
6-32 nuts (4 needed)	Ace Hardware		\$0.36 ²	1	\$0.36
SWITCH LEVER SPST 15A QC TERM	digikey.com	CH501-ND	\$4.51	1	\$4.51
Werker Spring-Top Heavy Duty Lantern Battery	batteriesplus.com	WKHD6VSPR	\$3.39	1	\$3.39
Undamped Magnetic Compass (package of 12)	USToy.com	1524	\$5.95	4	\$1.49
steel bar magnet	sciencekit.com	WW62285M00	\$6.50	1	\$6.50
Cen-Tech AC/DC digital multimeter	harborfreight.com	37772-2VGA	\$24.99	1	\$4.99
wire – 60 ft, 20 gauge	radioshack.com	278-1225	\$7.99	20	\$0.40
alligator clip lead (set of 8)	radioshack.com	278-1157	\$7.69	2	\$3.85
				TOTAL	\$42.61

- 1 Each screw costs \$0.25. The listed cost takes into account that two are needed for each apparatus.
- 2 Each nut costs \$0.09. The listed cost takes into account that four are needed for each apparatus.
- 3 This column represents how many copies of the apparatus can be made from the purchased item.
- 4 This is the cost per apparatus.