Exploring Students' Difficulties with Problems in Multiple Representations in Electromagnetism

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Overall Goals

Investigate:

- Common difficulties students encounter when solving physics problems in different representations.
- Hints that may to help students overcome these difficulties.

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Spring 2009 Study Revisited

- 4 Interview sessions with 20 engineering students taking Engineering Physics 1.
- Problems in mechanics.
- Numerical, graphical, and functional representations.
- Key Math concept: Integral equals area under graph.
- Effect of sequence of problems on students' performance

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Spring 2009 Study: Major Findings 1/2

- Students had difficulty in reading off and processing information from graphs to find the desired quantities.
- Students did not spontaneously recognize that integral was equal to the area under graph.
- Hints on mathematical or physical meaning were not as useful as those on basic issues such as units.

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Spring 2009 Study: Major Findings 2/2

- The sequence of the problems presented to students affected their performance.
  - Whether students were given the graphical problem or the functional problem first affected the average number of difficulties they had in an interview.
- Students seemed to gain representational competence as they progressed through our interviews.
  - Students had less difficulties working with graphs and functions in the later interviews.

Fall 2009 Study

- 4 Interview sessions with 15 engineering students (same as those in Spring) taking Engineering Physics 2.
- Problems in Electromagnetism.
- Numerical, graphical, and functional representations.
- Involve a variety of mathematical concepts and skills: differentiation, integration, geometric reasoning, …

Interview Problem Comparison

Spring Interviews	Fall Interviews
▪ 3 problems each interview (interview 1 has 2 problems).	▪ 4 problems in interview 1,3, 4, and 5 problems in interview 2.
▪ Based on exam problems.	▪ Based on homework problems.
▪ Each graphical problem has one graph – what to do with graph.	▪ Each graphical problem has 3 – 4 graphs – appropriate graph to use.
▪ Minor change in context (i.e. spring vs. gun), no change in geometry.	▪ Significant change in geometry.
▪ Probe basic understanding of the concepts/processes.	▪ Probe more deeply students' understanding and using of basic concepts/processes.

Research Design: Fall 2009

Interviews	Problem Sequences
Interview 1	$C_1R_1 \to C_1R_2 \to C_1R_3 \to C_2R_2$ $C_1R_1 \to C_1R_3 \to C_1R_2 \to C_2R_2$
Interview 2	$C_1R_1 \to C_1R_2 \to C_2R_1 \to C_2R_3$
Interview 3	$C_1R_1 \to C_1R_2 \to C_1R_3 \to C_2R_1$
Interview 4	$C_1R_1 \to C_1R_3 \to C_1R_2 \to C_1R_2$

C_1: One type of Geometry

C_2: Different type of Geometry

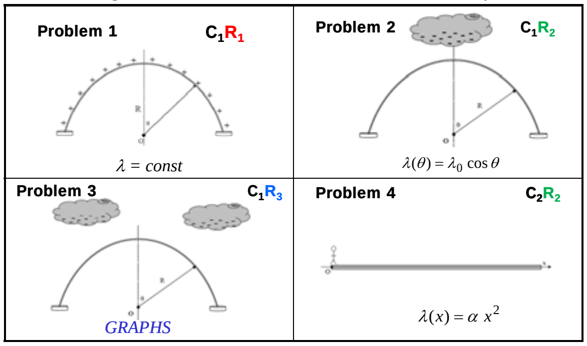
R_1: Numerical Representation

R_2: Functional Representation

R_3: Graphical Representation

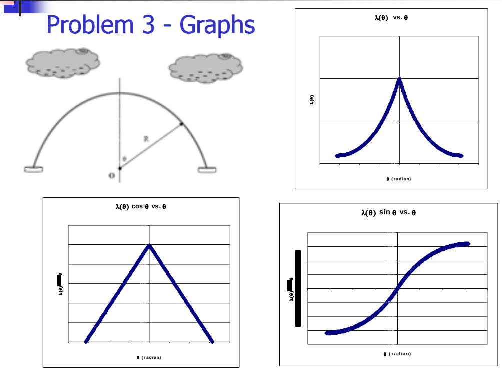
Interview 1 Problems

Find magnitude and direction of the electric field at point O.

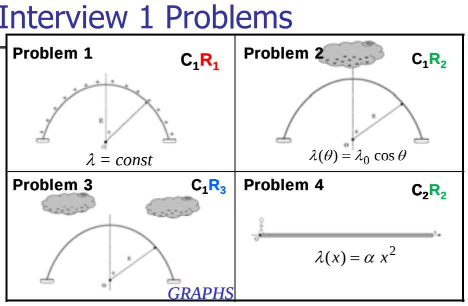


Interview 1 Problems

Problem 3 - Graphs



Interview 1 Problems



Issues regarding:

- Distribution of Charge
- Direction of Electric Field
- Magnitude of Electric Field

Interview 1 – All Probs. – General Impressions 1/7

Distribution of Charge

- Many students determine charge distribution based on the figure rather than from function or graphs.
- Some students have trouble in determining the sign of charges in Problem 2.
  - The change in definition of θ (down from the vertical) is part of the difficulty.
- Many students did not spontaneously mention the symmetry of the distribution in their verbal descriptions,
  - Although their drawings of the charge distributions were often symmetric.

Interview 1 – All Probs. – General Impressions 2/7

Distribution of Charge

- Students use a variety of strategies for indicating charge density:
  - varying the spacing between the charges
  - drawing different sized clumps of charge at equal spacing
  - drawing different size of pluses
  - drawing pluses under the graph of charge density function.

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Interview 1 – All Probs. – General Impressions 3/7

Direction of the Electric Field

- Most students were able to say that the electric field was vertically downward for Problems 1-3.
- Some students were able to talk about the horizontal components of the contributions from each side of the arch canceling.
  - Some other students made "this is what the professor did in class" types of explanations.
- The direction of the electric field in Problem 4 tended to be much more difficult for students.
  - Several students drew arched field vectors for Prob 4.

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Interview 1 – All Probs. – General Impressions 4/7

Magnitude of the Electric Field

- Several students wrote down/talked about the equation for Gauss' Law for finding the electric field (integrating **E.dA**).
- Most students knew that they needed a factor of cosine to pick out the vertical component of the electric field in problem 1.
  - However, in problem 2, several students didn't include this factor because the charge density itself already had $\cos\theta$.
- Some students have difficulty with switching between integration variables $dq = \lambda\, ds$.
  - Hints asking them to think about the definition of λ helped many students.

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Interview 1 – All Probs. – General Impressions 5/7

Magnitude of the Electric Field

- Nearly all students needed to be given the trig identity $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$ in Prob 2.
  - Many students were able to compute the integral with this information.
  - One student suggested that the integral of a product is the product of each function's integral.

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Interview 1 – All Probs. – General Impressions 6/7

Magnitude of the Electric Field

- All students had trouble deciding which graph to use for Problem 3.
  - Most students thought Graph 1 was the right one to use.
  - Several students wanted to use Graph 2 because the area was easy to calculate.
  - Discussion about how the integrand is related to the graph of a function whose area is the value of the integral helped most students.
  - Several students did not know what the ‘integrand’ meant.

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Interview 1 – All Probs. – General Impressions 7/7

Magnitude of the Electric Field

- Most students did not have trouble realizing they needed to integrate over θ for Problems 1-3, but several students didn’t know what ‘ds’ was in Problem 4.
- Most students referred to ‘r’ in Coulomb’s Law as radius. This caused them difficulties in problem 4.

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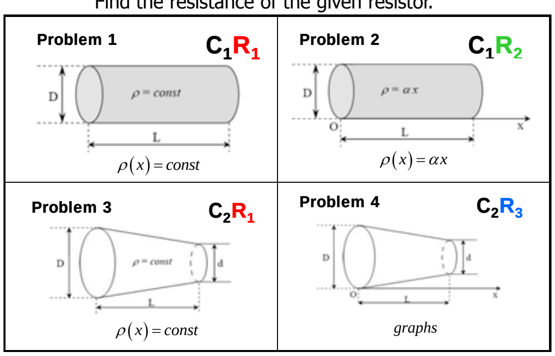
Interview 2 Problems

Find the resistance of the given resistor.



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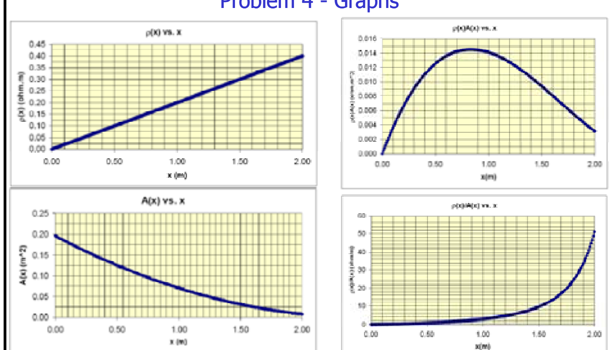
Interview 2 Problems

Problem 4 - Graphs



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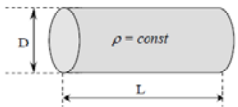
Interview 2 – General Impressions

Problem 1
Find the resistance of a cylindrical conductor of length L, diameter D and resistivity ρ (ρ is constant along the conductor).



$C_1R_1 \rightarrow C_1R_2 \rightarrow C_2R_1 \rightarrow C_2R_3$

- Do not remember formula for resistance.
- Thought of 'A' as surface area (with and without caps) or volume.

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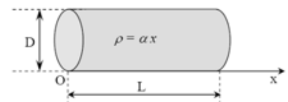
Interview 2 – General Impressions

Problem 2

Find the resistance of a cylindrical conductor of length L, diameter D. The resistivity $\rho(x)$ is changing along the conductor as per the following function:

$$\rho(x) = \alpha x$$

where x is the distance from the left end of the conductor.



$C_1R_1 \rightarrow C_1R_2 \rightarrow C_2R_1 \rightarrow C_2R_3$

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Interview 2 – Prob. 2 – General Impressions

- Integrated only the resistivity and multiplied by L/A.
  - Hinted by the unit of resistance.
- Knew but could not apply the meaning of integration.
- Needed help to recognize the meaning of 'dx' in the integral.

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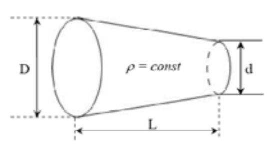
Interview 2 – General Impressions

Problem 3

A conductor has diameter decreasing from D to d over its length L. The resistivity ρ is constant along the length of this conductor. Find the resistance of this conductor.



$C_1R_1 \rightarrow C_1R_2 \rightarrow C_2R_1 \rightarrow C_2R_3$

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Interview 2 – Prob. 3 – General Impressions

- Knew that they had to integrate something, but were not sure what.
- Could not use geometric reasoning to find area of the resistor as a function of x.
  - Hinted by a graph of diameter vs. x.
- Some students thought limits of integral were from D to d because diameter was changing.
- Almost all students needed to be given the result of integral.
  - Only one student succeeded in using u-substitution to calculate the integral. Some students needed help adding/subtracting fractions.

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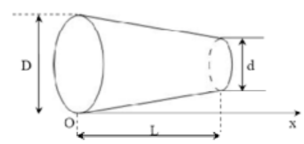
Interview 2 – General Impressions

Problem 4

A conductor has diameter decreasing from D to d over its length L. The resistivity of this conductor along the x axis is $\rho(x)$ and its cross-sectional area is $A(x)$. The graphs of $\rho(x)$ vs. x, $A(x)$ vs. x, $\rho(x)\cdot A(x)$ vs. x, and $\rho(x)/A(x)$ vs. x are given. Find the resistance of this conductor.



$C_1R_1 \rightarrow C_1R_2 \rightarrow C_2R_1 \rightarrow C_2R_3$

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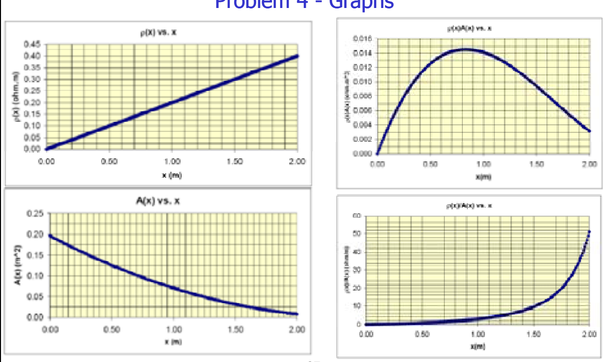
Interview 2 – General Impressions

Problem 4 - Graphs



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Interview 2 – Prob. 4 – General Impressions 1/2

- Most students tried to find function of $\rho(x)$ from the graph of to plug into the integral with function of A(x) known from problem 3.
  - The complicated integral forced students think of using area under graph.
- Some students claimed that integral of division of functions was division of each function's integral.

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Interview 2 – Prob. 4 – General Impressions 2/2

- Some students claimed to find area of graphs of ρ(x) vs. x and A(x) vs. x, then put those areas (numbers) into the integral.
- After such troubles as above, students were able to recognize that they should find area of the graph of ρ(x)/A(x) vs. x.

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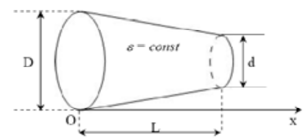
Interview 2 – General Impressions

Problem 5

A capacitor is made of two circular conducting plates of diameter D and d. The permittivity ε of the material filled between the plates is constant. Find the capacitance of this capacitor.



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Interview 2 – Prob. 5 – General Impressions

- Did not remember formula for capacitance of parallel-plate capacitor.
- Tried to set up an integral with 'dx' on the numerator.
- Did not spontaneously recognize series capacitors.
- Needed help converting sum to integral to find equivalent capacitance.

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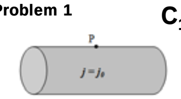
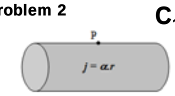
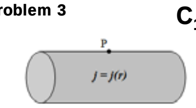
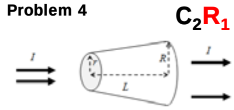
Interview 3 - Problems

Find magnitude of the magnetic field by the current at point P.

Problem 1 C_1R_1  $j(r) = j_0 = const$	**Problem 2** C_1R_2  $j(r) = \alpha r$
Problem 3 C_1R_3  *graphs*	**Problem 4** C_2R_1  $I = const$

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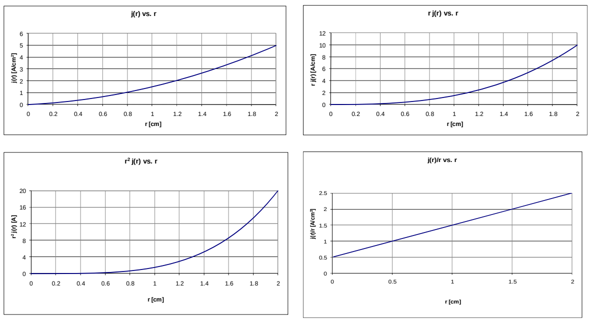
Interview 3 - Problems

Problem 3 - Graphs



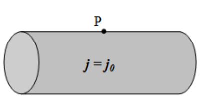
Interview 3 – Change in Problems

- Students did not know where to start and what to do to solve the problems.
- It was impossible to help students solve the problems without making the interview a tutoring session.
- Add a picture of the cross section of the wire.
- Split each problems 1 – 3 into two parts:
  - A) Find the total current in the wire.
  - B) Find the magnitude of the magnetic field at point P.

Interview 3 – General Impressions

Problem 1

A cylindrical wire of radius R is carrying a current of density $j = j_0$ (j_0 is a constant). Find the magnitude of the magnetic field caused by the wire at a point P on its surface.



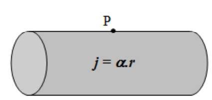
$C_1R_1 \rightarrow C_1R_2 \rightarrow C_1R_3 \rightarrow C_2R_1$

- Students asked for formula of current density, although they could figure it out themselves.

Interview 3 - Problems

Problem 2

A cylindrical wire of radius R is carrying a current of density $j = \alpha r$ (α is a constant, r is the distance from the center of the wire). Find the magnitude of the magnetic field caused by the wire at a point P on its surface.



$C_1R_1 \rightarrow C_1R_2 \rightarrow C_1R_3 \rightarrow C_2R_1$

Interview 3 – Prob. 2 – General Impressions

- Integrated j(r) only and multiplied by the total area.
- Students seemed to be so familiar with integrals with dx, dr, dθ, … that it didn't make sense to them to integrate **j.dA**,
  - Even though they remembered that, they failed to tell what dA meant.
- Students had difficulties finding dA.
  - Hints on derivative of area with respect to r helped.

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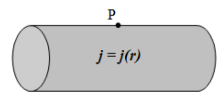
Interview 3 – Prob. 3 – General Impressions

Problem 3

A cylindrical wire of radius R = 2 cm is carrying a current of density *j* which depends on the distance *r* from the center of the wire as per the graphs given. Find the magnitude of the magnetic field caused by the wire at a point P on its surface.



$C_1R_1 \rightarrow C_1R_2 \rightarrow C_1R_3 \rightarrow C_2R_1$

- Some students confused between j(r) and **j.r**, so they thought of using the graph of j(r) vs. r.

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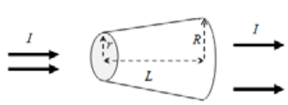
Interview 3 – Prob. 4

Problem 4

A tube carrying electric current expands uniformly over a distance *L*. The radius at the beginning of the tube is *r*, and at the end of the tube the radius is *R*. If the total current going through the tube is *I*, what is the average current density at location a quarter of the way down the tube (closer to the smaller end)?



$C_1R_1 \rightarrow C_1R_2 \rightarrow C_1R_3 \rightarrow C_2R_1$

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Interview 3 – Prob.4 – General Impressions

- Most students claimed to integrate the area, because area was changing.
- Needed help figuring out the function of diameter vs. x.

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Interview 3 – All 4 Probs. – Calculating **B** Field -- General Impressions

- Students tried to recall a formula for **B**.
- Hinted on Ampere's law and given its expression.
- Students had a hard time 'unwrapping' the left-hand side of Ampere's law.
- Some students wrote the left-hand side as B.2πR but failed (or used weak reasoning) to explain that result.
- Some students didn't know what 'ds' meant in the integral of Ampere's law.

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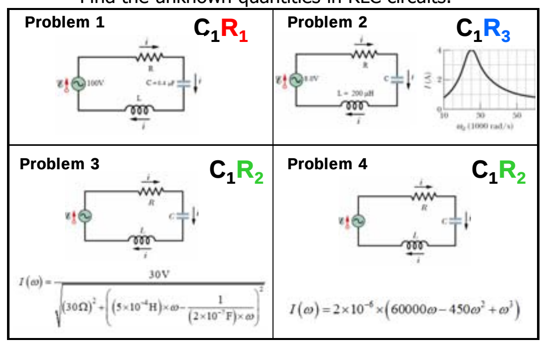
Interview 4 - Problems

Find the unknown quantities in RLC circuits.



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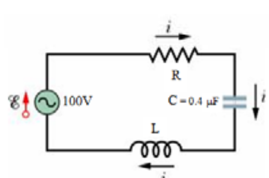
Interview 4 – Prob. 1

Problem 1

The current in a series RLC circuit reaches its maximum amplitude of $I_{max} = 2$ A when the driven angular frequency is $\omega_0 = 5\times10^4$ rad/s. The emf amplitude is 100V and the capacitance is 0.4 μF. Find R and L.



$C_1R_1 \rightarrow C_1R_3 \rightarrow C_1R_2 \rightarrow C_1R_2$

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Interview 4 – Prob. 1 – General Impressions

- Students either did not remember formulae or did not know that current depended on ω and reached maximum when $\omega = \omega_0$ (resonance).
- Most students seemed not familiar with the resonance case (did not know that $X_L = X_C$ and $I_{max} = E/R$ at resonance), so they could not simplify the problem.
- One student used the energy method to calculate L.

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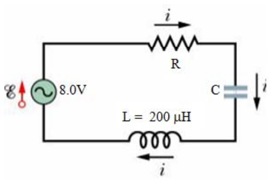
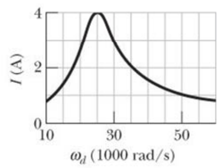
Interview 4 – Prob. 2

Problem 2

The current amplitude I versus driving angular frequency ω_d for a driven series RLC circuit is given in the graph below. The inductance is 200 μH and the emf amplitude is 8.0 V. Find C and R.





$C_1R_1 \rightarrow C_1R_3 \rightarrow C_1R_2 \rightarrow C_1R_2$

Interview 4 – Prob. 2 – General Impressions

- Most students chose the point of maximum current on the graph, but could not explain their choice.
- Some students did not know which point to choose or chose a random point on the graph to get I and ω.

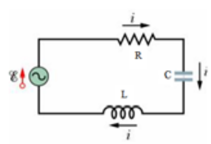
Interview 4 – Prob. 3

Problem 3

The current amplitude I(ω) (in Amperes) of a series RLC circuit depending on the driving angular frequency ω (in radian/second) is given as follow:

$$I(\omega) = \frac{30\text{V}}{\sqrt{(30\Omega)^2 + \left((5\times10^{-4}\text{H})\times\omega - \frac{1}{(2\times10^{-7}\text{F})\times\omega}\right)^2}}$$

Find the resistance R, inductance L, capacitance C, resonance frequency ω_0, and maximum current amplitude I_{max}.



$C_1R_1 \rightarrow C_1R_3 \rightarrow C_1R_2 \rightarrow C_1R_2$

Interview 4 – Prob. 3 – General Impressions

- The mapping task in this problem was very easy for all students.
- The units of quantities helped them to some extent.

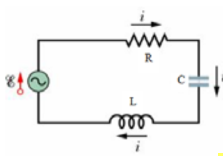
Interview 4 – Prob. 4

Problem 4

The current amplitude I(ω) (in Amperes) of a series RLC circuit depending on the driving angular frequency ω (in radian per second) is given as per the following function:

$$I(\omega) = 2\times10^{-6}\times\left(60000\omega - 450\omega^2 + \omega^3\right)$$

Find the resonance angular frequency ω_0 and the maximum current amplitude I_{max}.



$C_1R_1 \rightarrow C_1R_3 \rightarrow C_1R_2 \rightarrow C_1R_2$

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Interview 4 – Prob. 4 – General Impressions 1/2

- Since students were not familiar with the resonance case, they did not know how to do this problem.
- When hinted that they needed to find ω that made I(ω) maximum, they still could not think of the mathematical process to solve the problem.
- Hint on an analogous mathematics problem of finding value of x that made f(x) maximum was helpful to some students but not to others.

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Interview 4 – Prob. 4 – General Impressions 2/2

- The first few students were told that a function reached extreme values at zeros of its first derivative.
- Other students were given a graph with maximum and minimum, and asked to find the common property of those points.
- Students found two zeros of first derivative of I(ω). Some thought that the larger ω gave larger current, others plugged each ω into I(ω) to find current and compared.
- Hinted on the change of slope when passing the maximum and minimum points.
- Only two students (out of 15) mentioned the "second-derivative test".

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Fall '09 – All 4 Interviews – Overall Impressions

- "Integral = area under graph" seemed to be obvious to students, but they had difficulties choosing the right graph to use.
  - Thinking more deeply on the relation between graph and integral, they no longer chose a graph because it was easy to find area.
- Students knew meaning of mathematical operations (derivative, integration, …) but could not apply that knowledge in the problems.
- Students seemed to automatically integrate anything that was changing.

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Summary of students' difficulties in each representation

Number	Picture	Function	Graph
Algebraic computation	Associating symbols with the picture	Algebraic computation	calculus processes for finding features:
Associating symbols with quantities	Writing down a function from the picture:	Calculus computation	integral -> area
Units	Ex: Area as a function of distance	Appropriate application of formulae for special cases: If f(x) = kx^2, then U(x) /= 1/2 kx^2)	derivative -> slope
		Identifying integration variable	finding f_max
		Units	estimating area/slope
			units of area/slope

Next Steps

- Phenomenographic analysis of transcripts.
- Investigate resources that students activated to solve the problems and the factors that affected their choice of resources.

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Spring 2010 Plans

- More detailed literature review on multiple representations.
- Focus group interviews?
  - E.g. Similar to Fran's Interviews
- Framework for explaining results: Candidates
  - Conceptual Resources (Hammer)
  - Cognitive Framework for Math in Phys (Tuminaro)
  - Dynamic Transfer (Schwartz)

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Thank You

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Comments

- Why did only 1 student think integral of product was product of integrals while several thought that for quotients?
  - Literature suggests that when students find questions difficult to comprehend the conceptual level of all their work can suffer.
- Can you relate errors by individual students across the interviews?
  - Take advantage of the longitudinal structure of your work to see how conceptions and misconceptions develop in individual students.
- You need a framework to allow you to trace conceptual growth over time
  - But you note this in your future plans.