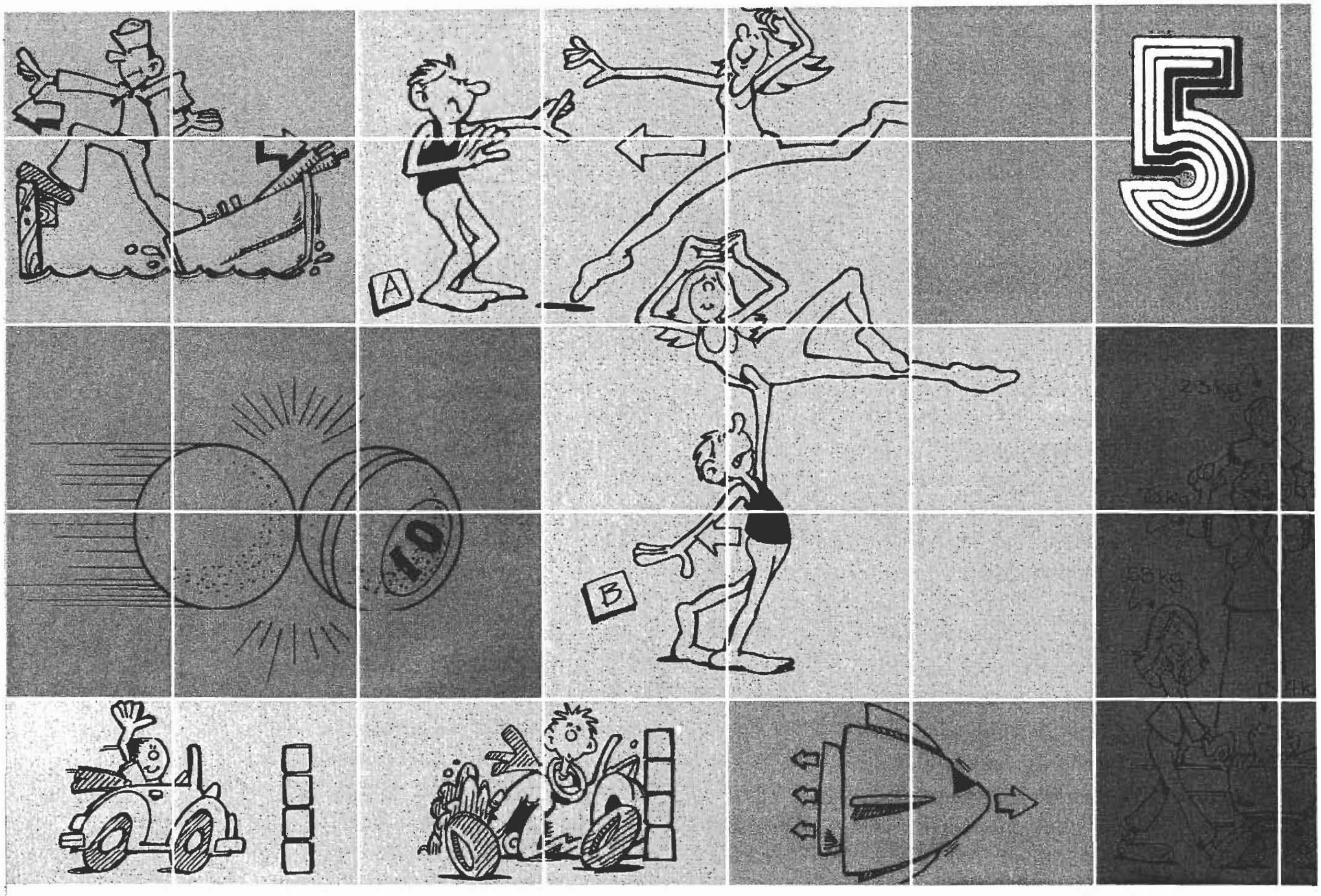


Interaction and Momentum

I woke to myself and looked about me, and said to the folk of Troizen, "I have had the sign of Poseidon. He will shake the earth and soon. Warn them in all the houses to come out of doors. Send word to the Palace."

Theseus, King of Athens

Geologists can demonstrate that at least eight major earthquakes have occurred (on the southern San Andreas fault) in the past 1200 years with an average spacing in time of 140 years, plus or minus 30 years. The last such event occurred in 1857.... The aggregate probability for a catastrophic earthquake in the whole of California in the next three decades is well in excess of 50 percent.

Federal Emergency Management Agency

Change, whether abrupt like earthquakes or gradual like aging, is our constant companion. For Greek and Roman civilizations, change belonged to the gods. When displeased or angered, Poseidon shook the earth, Zeus hurled thunderbolts, and the Furies exacted punishment. A host of gods and goddesses governed all of life. Men like Theseus, who received signs directly from a god, or priests and priestesses skilled at interpreting oracles predicted the actions of the gods.

Our present model of earthquakes explains them in terms of the motion of crustal plates, which alternately stick and slip along their boundaries. Greece, the land of Theseus, lies along one such boundary; California, along another. Patterns of past earthquake activity and continual measurements of the motion of the plates are beginning to allow scientists to make rough predictions of future earthquakes. For science, change is a natural process. By observing carefully, we can build models or explanations that eventually allow us to predict change. Physicists describe change in terms of *interaction.* This chapter introduces the concept of *momentum* and describes its use in explaining interactions.

INTERACTIONS

When one object influences another, we say that the two objects *interact.* In order to analyze an interaction, we must be able to see some change. Fallen houses and displaced trees tell us that two crustal plates have interacted along their boundaries. The building of mountain ranges and eruption of volcanoes is evidence for the slow collision of continents brought about by the movements of plates over millions of years. **Change,** then, provides us the *evidence that an interaction* occurred. Measurements of the amount of change enable us to see patterns and build models that predict future change.

Because the details of an interaction can be extremely complex, we often look for change simply by comparing the "before" with the "after." To see how this works, compare before with after for each situation in Figure 5-1. The pictures in (a) show a standard advertising gimmick: "Interact with our product and you'll see a change!" A change in shape like (b) implies an interaction-presumably an unwanted one.-Another change can be a change in velocity, such as that experienced by the ball in (c). An enormous variety of changes are possible, including changes in shape, size, volume, velocity, and temperature, to name a few. Simply comparing the "before" with the "after" enables us to identify and categorize these interactions.

In this chapter we will restrict ourselves to interactions in which a change in velocity has occurred, such as that shown in Figure 5-1(c). If an object slows down, speeds up, or changes direction, it must have interacted with something. You might ask why we have linked interactions with changes in velocity, since motion at a constant velocity also involves change-a change of position. But, objects that are stationary in one reference frame can be moving to observers in other reference frames, as you saw in Chapter 3. A change in position can occur when no interaction has occurred-the observer is simply

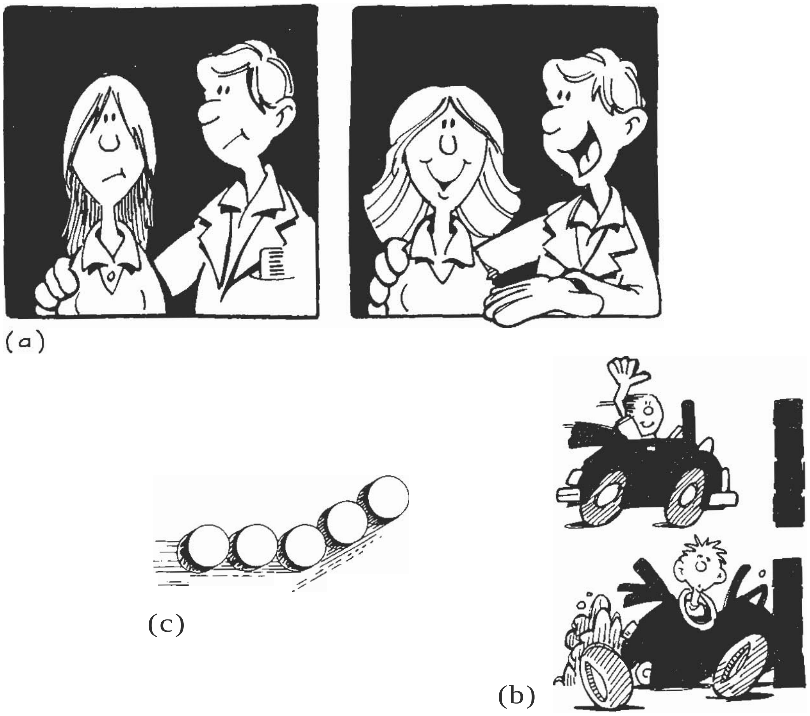


Figure 5-1
A change provides evidence that an interaction has occurred.

in a reference frame that is moving relative to the object. A change in velocity, however, does imply an interaction. We begin by looking at changes in velocity that occur when two objects interact.

SELF-CHECK 5A

Which pair(s) of before and after sketches of the ball in Figure 5-2 show(s) evidence that an interaction has occurred?

FACTORS AFFECTING INTERACTION

A tennis ball hitting a net, an egg striking the floor, a car colliding with a tree-in each case the motion of the object is abruptly stopped. An interaction has occurred. Yet tennis balls, eggs, and cars can experience other interactions that are not quite so abrupt. To understand interactions and the way in which they influence motion, we need to identify the characteristics of objects that affect their interactions.

Velocity Affects Interactions

A friend lobs a baseball, which you catch with your bare hand. It is an easy catch-you hardly feel it. Now your friend throws a fastball. If you catch it,

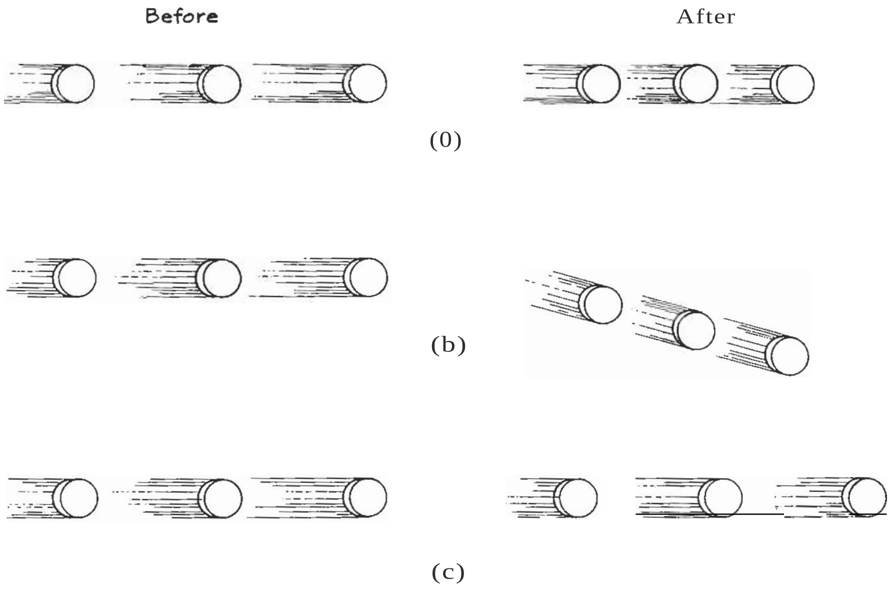


Figure 5-2

your hand stings; the speed of the ball affects the way your hand feels. Much the same thing happens when an automobile collides with a tree. An automobile moving 10 kilometers per hour (km/h) will not damage the tree nearly as much as an automobile moving 60 km/h. An egg dropped 1 centimeter (cm) is not moving as fast as an egg dropped 1 meter (m). The egg's speed before it interacts with the floor determines whether or not the egg breaks.

The *speed* of the object is not the only variable that affects the interaction. *Direction* is also involved. In automobile accidents, head-on collisions are much more damaging than rear-end collisions. Your hand will sting less if you move it away from the ball rather than toward it as you catch it. In the extreme case, the egg would not even interact with the floor if the egg were moving upward rather than downward. *Velocity,* which combines the concepts of speed and direction, affects the interaction.

Mass-A New Quantity

Velocity by itself does not explain all the differences that we see in interactions. A tennis ball and a hardball, both thrown at the same velocity, will leave markedly different impressions on your hand. A car is stopped by its collision with a tree, while a large truck may be only momentarily slowed as it knocks the tree down-even when the initial velocities of the car and truck are the same. The difference between a tennis ball and a hardball or between an automobile and a truck is the amount of matter each has.

The concept we use to describe the amount of matter in an object is its mass. Intuitively, we define **mass** as a measure of the amount of matter. To measure it, we establish standards and compare unknown masses to these standards. The fundamental unit of mass is the kilogram (kg). The mass of a tennis ball is about 0.064 kg, while the mass of a baseball is about 0.142 kg. Cars and trucks have an even larger range of masses-1000 kg for a small

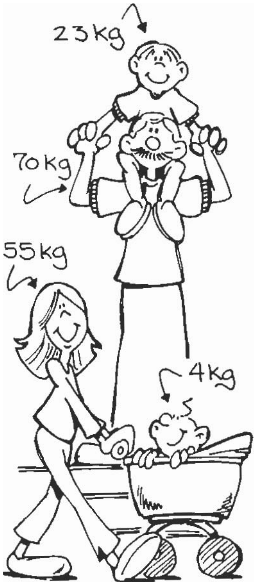


Figure 5-3
The range of human masses is from a few kilograms to well over 100 kilograms.

car and 10,000 kg for a truck. Typical human masses are illustrated in Figure 5·3. Since direction has no meaning in describing the amount of matter in an object, mass is a scalar quantity.

In everyday conversation we use the terms *mass* and *weight* interchangeably. We say that the weight of a loaf of bread is half a kilogram, though we have really described the bread's mass. As you will see in Chapter 6, weight and mass have distinct meanings in physics. Mass refers to the amount of matter in an object, while weight describes the strength of the interaction between the object and the planet on which it is located. While weight does depend on the object's mass, it also depends on characteristics of the planet. Consequently, an object's weight is different on the moon than on the earth. An object's mass remains constant throughout space, while its weight varies with its location. Because we want to develop models that apply in space as well as on earth, we use mass to describe the effect that the amount of matter has on interactions.

SELF-CHECK 58

Which variable, velocity or mass, affects these interactions?

a. You play catch with a basketball rather than a bowling ball.

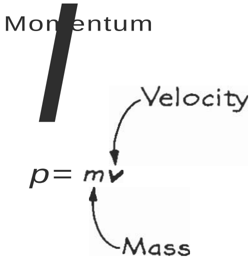
b. In defense practice a football coach has the players tackling from in front of rather than behind the runners.

c. Your neighbor throws two identical eggs at a wall. One barely breaks; the other shatters completely.

MOMENTUM

Two variables, mass and velocity, help describe the different interactions we observe. Consider the role of each by using your hand to judge the strength of interaction with tennis balls and baseballs. You can mix the two variables, mass and velocity, in four ways: *low mass, low velocity; low mass, high velocity; high mass, low velocity;* and *high mass, high velocity.* From experience you can probably identify the extremes. A tennis ball lobbed toward you (low mass, low velocity) will sting very little compared to a baseball hurled by a fastball pitcher (high mass, high velocity). More difficult to judge is the difference between a tennis ball hurled by a pitcher (low mass, high velocity) and a baseball lobbed gently (high mass, low velocity). The reason it is more difficult to judge is that the two variables-mass and velocity-are actually combined in our perception of the interaction. In physics the concept of momentum combines the concepts of mass and velocity.

Momentum Defined



Momentum is defined as the product of mass and velocity:

momentum = (mass) X (velocity)

Mass is a scalar quantity. Velocity is a vector quantity. Consequently, momentum is defined as a vector quantity whose direction is the same as the direction of the velocity. The units in which momentum is measured are the units given by its definition: kilogram-meters per second (kg. *m/s).*

In one sense, momentum is a measure of the "influence" one object has on another in an interaction. We can explore this idea by calculating the momentum of each ball just before it interacts with your hand. The mass of a tennis ball is 0.064 kg and that of a baseball is 0.142 kg. We can estimate the speeds of a lob and a fastball to be 5 *mls* and 50 *mis*, respectively. Given these values, the momentum of the tennis ball lobbed toward you is (0.064 kg)(5 *mis*, east) = 0.32 kg . *mis*, east. With its larger mass, a baseball lobbed toward you has a momentum of (0.142 kg)(5 *mis*, east) = 0.71 kg . *mis*, east. A tennis ball hurled toward you has a still larger momen· tum, (0.064 kg)(50 *mis*, east) = 3.2 kg . *mis*, east. Finally, a baseball hurled toward you has the largest momentum, (0.142 kg)(50 *mis*, east) = 7.1 kg . *mis*, east. The ordering of momenta (plural of momentum) from least to most agrees with the feeling you have about how each one would hurt as it struck your hand. Moreover, the quantitative definition of momentum allows us to distinguish subtly different descriptions from one another-for example, low mass, high velocity from high mass, low velocity.

SELF-CHECK SC

A ping-pong ball lies motionless on a table. Each of the balls listed below rolls toward and collides with it. Which has the most momentum? Which do you thfnk will cause the greatest change in the motion of the ping-pong ball?

	Mass	*Velocity*
Glass ball	0.10 kg	1 m/s, E
Super ball	0.12 kg	0.5 m/s, E
Lead ball	0.20 kg	0.2 m/s, E

Momentum Before and After an Interaction

We can calculate a momentum for any object, even if that object does not interact with other objects. As you might expect, these kinds of calculations are

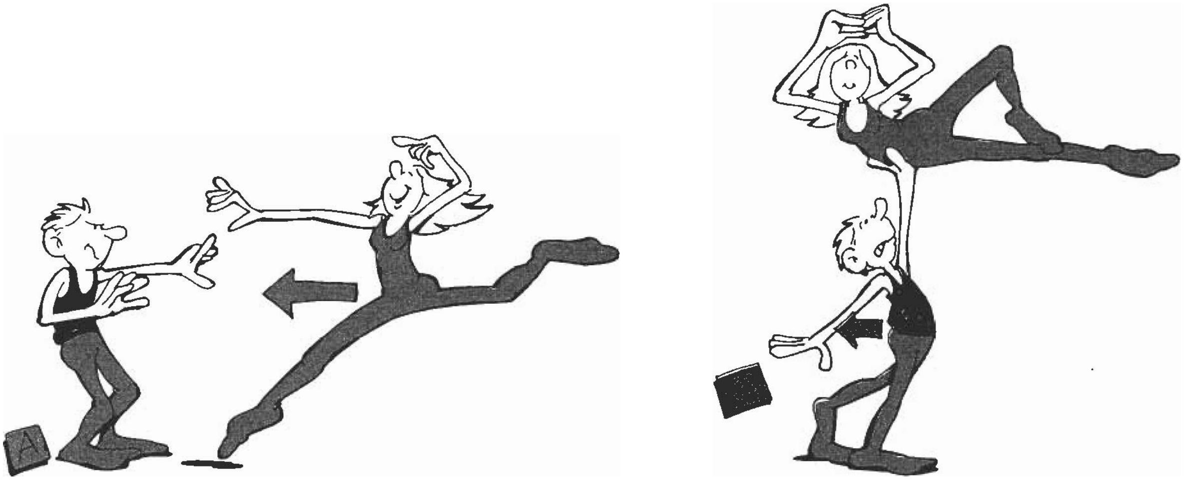


Figure 5-4
After the interaction the second dancer steps back with a velocity less than that of the first dancer before the interaction. Her momentum before their interaction equals their total momentum afterwards.

not always particularly interesting. But when one object interacts with another, momentum becomes a very useful concept; it is the key that enables us to predict the outcome of the interaction.

To examine the role of momentum in predicting the outcome of interactions qualitatively, consider an example taken from classical ballet (Figure 5-4). One dancer runs toward the second, who is initially standing still. The second dancer catches the first and moves several steps backward in the process. The second dancer does not move backward with the same speed as the first; his speed is always less. Before the interaction, the first dancer has a momentum equal to the product of her mass and velocity. The second dancer has no momentum; his velocity is zero. After the interaction, the couple has a momentum equal to the sum of their masses times the velocity with which the second dancer steps backward.

If you compare the situation before with the situation after, momentum provides the link. The direction of motion is the same before and after the interaction-the dancers always move to the left. Greater mass compensates for the lower speed with which the second dancer steps backward, because he carries the first dancer. The momentum of the two dancers together immediately after is about the same as the momentum of the first dancer before the interaction occurred.

Momentum: A Constant of Motion

The example drawn from classical ballet actually involves interactions among more objects than just the two dancers. Both dancers interact continually with the floor, the air about them, and the earth. In addition, their motion is not simply horizontal, since the second dancer lifts up his partner. These additional interactions complicate our analysis and make it difficult to conclude anything more than that momentum seems to be involved in describing the outcome of the interaction.

Devices such as air hockey tables remove many of these complications. Consider an experiment with two air hockey pucks that can stick together upon impact. (Matching strips of velcro attached to each air hockey puck work well.) One lies essentially motionless, like the second dancer. The second puck moves toward the first, collides with it, and sticks to it. The two pucks then move backward, just as the second dancer stepped backward after lifting the first dancer. Both air hockey pucks float on a cushion of air, so their interactions with the table are minimized. Their motion is restricted to just one plane-the flat surface of the table. Thus, we can look at an experiment analogous to the two dancers, but one that minimizes the effects of all interactions except the interaction between the two pucks.

Figure 5-5 shows a strobelike drawing of the motion both before and after the interaction. The first puck, with a mass of 0.170 kg, is stationary. The second puck, also with a mass of 0.170 kg, has a velocity of 10 meters per second *(m/s)* to the right. After the interaction, the two pucks stick together and move on to the right. Their combined mass is 0.340 kg. Their velocity is 5 *mls* to the right-half the initial velocity of the second puck. Using this information, we can compare the momentum before the interaction with the momentum after the interaction:

Momentum before = (0.170 kg)(10 *mis*, right)
= 1.7 kg . *mis*, right

Momentum after = (0.340 kg)(5 *mis*, right)
= 1.7 kg . *mis*, right

The two momenta are the same.

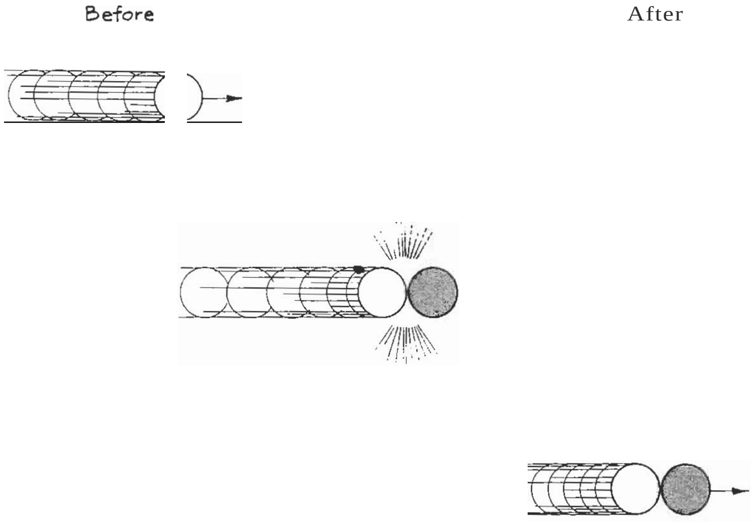


Figure 5-5

The second puck moves at a velocity of 10 *mIs*, right, toward a stationary puck. They stick together and move off at a common velocity of 5 *mIs*, right. With twice the mass, the combined pucks move at half the initial velocity of the single puck.

Momentum links the action before with the action after. Of all the results we can imagine for any given interaction, the result we actually see is that which keeps momentum constant.

SELF-CHECK 5D

You are standing motionless on a frozen pond. A friend whose mass is 70 kg slides toward you at a velocity of 2 m/s, east. The friend grabs you and the two of you slide oIUogether. What is your friend's momentum before the interaction? What is the momentum of the two of you after the collision? Describe your motion after the collision:

CONSERVATION LOGIC

The notion that some quantities are constant throughout interactions is one that is deeply embedded in our view of everyday life. If you cannot find your house key, you search for it. You assume the key still exists-that it has not vanished into thin air. The continued existence of the key is something you take for granted. Yet, studies about how people develop intellectually show that you were not born with this sense of the constancy of things. The idea that certain things remain the same, which we call *conservation logic,* emerged from your experience with the world.

Ideas of Constancy Develop with Age

After years of careful study, researchers such as Jean Piaget have concluded that infants have no sense of constancy. Babies younger than about 6 months behave as though objects that are not visible do not exist. If you hold a favorite toy in front of a very young child, the baby will smile and reach for it. If you then hide the toy under a blanket as the child watches, the child immediately begins to cry. For the infant, the toy no longer exists. With experience, children form a concept of constancy of objects. Older babies will actively search for the toy, trusting that it continues to exist throughout the action of hiding. As we grow older, this sense of constancy broadens to include a number of other quantities.

Two simple tests often used to assess the reasoning skills of young children involve our concepts of substance and number. In one, a ball of clay is rolled into a long, thin snake as the child watches. The child is asked whether the snake has more clay, less clay or the same amount of clay as the ball. In a second test, three pennies are placed side by side on a table. As the child watches, the pennies are moved farther apart. The child is asked whether the second configuration has more pennies, fewer pennies or the same number of pennies as the first.

These questions may seem rather silly to you, but to a child they are not. Young children (4-6 years old) frequently respond that there is more clay in the snake and that there are more pennies in the second configuration. With experience, they gradually come to realize that the amount of clay and the number of pennies remain constant throughout the interactions.

Both of these exercises examine the development of conservation principles. A **conservation principle** states that a quantity does not change as a result of certain interactions. The amount of clay remains constant during the interaction with your hand-this is called **conservation of substance.** The number of pennies remains constant during changes in spacing-this is called **conservation of number.** In studying interactions, we examine yet another conservation principle, conservation of momentum.

Systems and Conservation Logic

Conservation principles are valid only when we are careful to define the objects involved in the interaction. Conservation of substance, as illustrated with the clay, is valid only when we do not add or take away any clay. Conservation of number remains valid only when pennies are not added or subtracted.

Another example is conservation of mass. Generally we expect the operation of sawing a board into two pieces to have no effect on the mass of the board; that is, the mass of the board is conserved. But if we compare the mass of the original board before with the combined mass of the two pieces after, we find mass is not conserved. Why? Because of the sawdust on the floor. Once the mass of the sawdust is included with the mass of the two pieces, mass is indeed conserved. Conservation principles are valid only when we are careful to keep track of all objects involved in the interaction.

The concept of a *system* helps identify and keep track of objects. A **system** is any set of objects we wish to study. Once the system has been identified, interactions can be divided into two groups:

1. interactions between objects in the system; and
2. interactions between an object in the system and an object outside the system.

In Figure 5-5 we defined our system to be the two air hockey pucks. When the two pucks collide with each other, an interaction occurs between objects within the system. If one puck hits the edge of the table, the interaction involves an object outside the system. When we choose our system so that all the interactions we are concerned with occur between objects in the system, we have chosen a **closed system. The** pucks compose a closed system as long as they interact only with each other.

Conservation principles are valid only in a closed system. Momentum is conserved in the closed system made up of the two air hockey pucks. Once the pucks bounce off the sides of the table, they have interacted with an object outside the system, and the momentum of the pucks will not be conserved. If we were to define a larger system that includes the two pucks and the table, momentum would be conserved for all objects in the system. But then we would have to include the mass and motion of the table (though its motion would be very minute) in addition to the mass and motion of the two pucks in our analysis. One way to be sure that you can apply conservation principles is to define your closed system to be the entire universe. But this presents us with a different problem-it is impossible to keep track of all the objects in the universe and most objects are not relevant to a given interaction anyway. (The snow in Moscow does not affect a basketball game in Poughkeepsie.) That is why we limit ourselves to the objects actually involved in the interaction.

As we gain experience with objects, our list of quantities that are conserved grows. Most of these conservation principles, like conservation of mass and number, become commonsense and we're surprised to learn that we didn't recognize them from birth. Others, like conservation of energy and momentum, we may realize at an intuitive level but don't verbalize until we study a specific field, like physics. Still others have yet to be discovered.

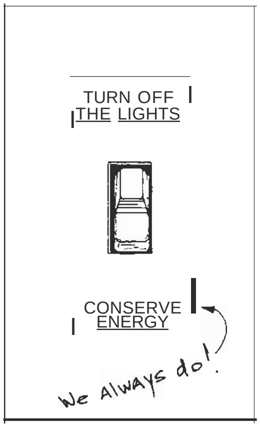


Figure 5-6

WARNING

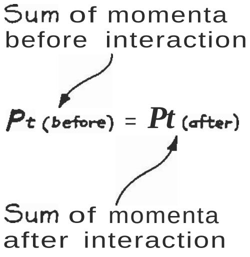
In normal usage (especially today!), to conserve means to save. In physics, to conserve means to keep constant. Confusing *conserve* in everyday language with *conserve* in physics may be hazardous to your understanding.

CONSERVATION OF MOMENTUM

While most of us have some intuitive sense of momentum, applying conservation of momentum often seems rather formal. Choosing a closed system, identifying the objects that interact, separating the action before from the action after, determining the momentum of the objects involved in the interaction, finding the total momentum of the system-these steps are all part of the process of using conservation of momentum to predict the outcome of interactions. Let's look at how this process works.

Principle of Momentum Conservation

The **principle of momentum conservation** states that in any closed system, the total momentum of the system does not change even though objects within the system interact with one another. We can express this principle in equation form as



$$(\text{total momentum})_{\text{before}} = (\text{total momentum})_{\text{after}}$$

If the closed system consists of just two objects, this becomes

$$(\textbf{momentum } 1 + \textbf{momentum } 2)_{\text{before}} = (\textbf{momentum } 1 + \textbf{momentum } 2)_{\text{after}}$$

If the closed system involves more than two objects, we simply add more terms, one for each object.

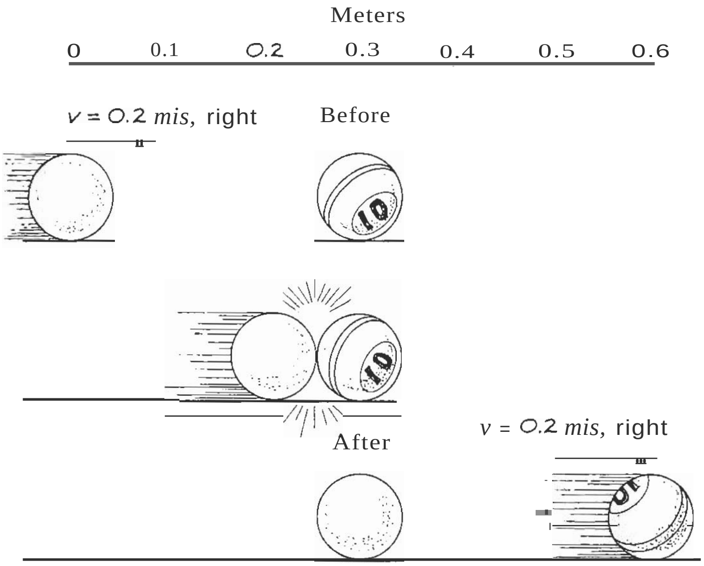


Figure 5-7

In this interaction, the cue ball transfers its momentum to the 10-ball. The cue ball moves in; the 10-balJ moves off.

In applying this principle, it is important to distinguish between the momentum of individual objects in the system and the total momentum of the system. As objects interact, their individual momenta certainly do change. The momentum of each of the two dancers after their interaction was different from before. It was the *sum of their momenta* that remained constant before, during, and after the interaction. If, in applying the principle of momentum conservation, we find that the total momentum is not conserved, we conclude that other objects have been involved-that our system was not closed.

Collisions

When an air hockey puck collides so that it sticks to an identical puck, we see an example of momentum conservation. When the two pucks stick together, the mass of the moving object doubles. Consequently, the two pucks move off at a speed that is one-half the initial speed of the first puck alone. The momentum before the collision equals the momentum after the collision.

Momentum is also conserved when objects do not stick together. A common example of this type of collision is the interaction between the cue ball and another ball during a game of billiards. When the cue ball is shot without spin, it strikes another ball and stops. The second ball then moves off, as shown in Figure 5-7. In this type of collision, the object moving before the interaction is not the object moving after the interaction.

Let's apply the principle of momentum conservation to a closed system consisting of two billiard balls-the cue ball and the la-ball. The mass of the two balls is identical, 0.170 kg. Before the interaction, the cue ball has ave·locity of 0.2 *mis*, right. The la-ball is motionless. After the interaction, the cue ball is motionless and the la-ball moves away at a velocity of 0.2 *mis*,

Figure 5-8

The momentum of the system was zero before the interaction. As the person steps forward, the boat must move backward with a momentum equal to the person's forward momentum.

right. Expressed in terms of equations,

momentum before = {0.170 kg}{0.2 mfs, right} + {0.170 kg}{O mfs}

= 0.034 kg . mfs, right

momentum after = {0.170kg}{Omfs} + {0.170kg}{0.2mfs, right}

= 0.034 kg . mfs, right

The momentum of the system remains constant during the interaction. The momentum of the cue ball is simply transferred to the lO-ball.

We have treated interactions of billiard balls ideally-that is, we have pretended that no interactions occur except the ones that interest us. In reality the billiard balls interact with the table, thus introducing an interaction outside the closed system. Additionally, billiard players do not simply roll the cue ball toward the 10-ball; they use a variety of spins to control the motion of the cue ball after the interaction. Essentially though, good billiard players develop an intuitive sense of momentum conservation in both one and two dimensions.

Stepping out of a Rowboat and Steering a Spaceship

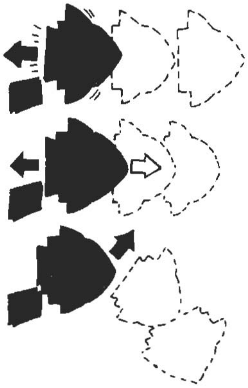


Figure 5-9

Spaceships use conservation of momentum in the design of their steering and control mechanisms. **(8)** When gas is ejected out the back, the spaceship speeds up. **(b)** When gas is ejected out the front, the spaceship slows down. **(c)** When gas is ejected upward, the spaceship moves downward.

A more subtle example of momentum conservation occurs when you try to step out of a rowboat {Figure 5·8}. Again, our system involves just two objects-you and the rowboat. {We assume that the boat's interaction with the water can be ignored.} Initially, you are standing in a stationary rowboat. Assume that your mass is 60 kg and the mass of the rowboat is 40 kg. Since neither you nor the rowboat is moving initially, the total momentum of the system is zero. Conservation of momentum tells us that the total momentum of the system as you step out must still be zero. If you move forward at a speed of 1 mfs, your momentum is 60 kg . mfs, forward. In order for the total momentum of the system to remain zero, the boat must move with a momentum of 60 kg . mfs, backward. Your forward momentum must be canceled by the boat's backward momentum.

Spaceship designers take advantage of this when developing steering mechanisms. On earth we speed up, slow down, or change direction by interacting with the ground. In space, pilots use small gas outlets placed along the outside of the spaceship {Figure 5·9}. To change velocity, the pilot ejects a large number of high-speed gas molecules from a selected outlet. Each molecule has a small momentum. Like the boat that moves backward as you step forward, the spaceship gains a momentum equal to but in the direction opposite to the ejected gas molecules. If the pilot wants to speed up, he or she fires the gas in a direction opposite to the present motion. To slow down, the pilot fires the gas in the same direction as the spaceship is moving. Other orientations will turn the vehicle.

SELF-CHECK 5E

A loaded **rifle is initially motionless. The trigger is pulled and a** 0.02 kg bullet **leaves the rifle** with a **velocity of 300 m/s, N. What is the velocity of the** 10 **kg rifle after the bullet is fired?**

A STEP FURTHER-MATH

A VERY STUBBORN EARTH

Momentum conservation allows us to do more than say that the boat moves backward with a momentum of 60 kg . *m/s*. If we write the equation that describes momentum conservation,

$$\mathbf{p}_{T(\text{before})} = \mathbf{p}_{T(\text{after})}$$

$$0 = m_{\text{you}}\mathbf{v}_{\text{you}} + m_{\text{boat}}\mathbf{v}_{\text{boat}}$$

we can rearrange it to solve for the velocity of the boat.

$$m_{\text{boat}}\mathbf{v}_{\text{boat}} = -m_{\text{you}}\mathbf{v}_{\text{you}}$$

$$\mathbf{v}_{\text{boat}} = -\frac{m_{\text{you}}}{m_{\text{boat}}}\mathbf{v}_{\text{you}}$$

Your mass is 60 kg and the mass of the boat is 40 kg. If you step forward with a velocity of 1 *mis*, forward, then the velocity of the boat is

$$\mathbf{v}_{\text{boat}} = -\frac{60\text{ kg}}{40\text{ kg}}(1\ \textit{mis}\text{, forward})$$

$$= 1.5\ \textit{mis}\text{, backward}$$

You step forward at a speed of 1 *m/s;* the boat moves backward at a speed of *1.5 m/s*. Why faster? Because the boat is less massive than you.

Take a step on land. Does the earth move backward as you step forward? You bet it does-but it is a bit harder to notice. Substitute the earth for the boat in the equation we just derived. The earth's mass is about 6 X 10^{24} kg. If you step forward with a speed of 1 *mis*, how fast does the earth move backward? How about when you and a friend step forward in the same direction? How about when you and a thousand friends step forward in the same direction? It's a mighty stubborn earth!

INTERACTIONS WITH LARGE MASSES

Conservation of momentum is easy to notice when the two interacting objects have about the same mass. We have no difficulty recognizing it in interactions between billiard balls or collisions between air hockey pucks. But when the two objects differ greatly in mass, conservation of momentum is no longer obvious. A person catching a tennis ball, an egg striking the floor, an automobile colliding with a tree-in each case, the system had momentum before the interaction. After the interaction, however, nothing *appears* to move. The momentum of the system *seems* to have disappeared.

One response to this dilemma would be to suggest that momentum simply is not conserved in these situations. However, conservation logic has been such a powerful tool in understanding interactions that physicists are unwilling to abandon it. Instead, they look for an explanation in terms of momentum conservation. We now examine their explanation by first considering a series of ideal interactions between train cars.

Increasing the Mass Decreases the Velocity

Figure 5-10 shows what happens before and after collisions between two identical train cars when one of them is loaded with successively heavier masses. In each case, car X is moving initially, while car Y is stationary. When they collide, the two cars lock and move off to the right as a single unit. The drawings in (a) show the motions before and after a collision between two equal (empty) cars, while (b)-(e) show the motions as successively heavier loads are added to car Y. Consequently, in each drawing, car X interacts with a larger mass than in the previous drawing.

In each collision, the closed system consists of two train cars and any cargo we place in car Y. Before the collision, the momentum of the system is the momentum of car X. This quantity is the mass of car X times the velocity of car X. After the collision the momentum of the system is the total mass (car X + car Y + cargo in car Y) times their common velocity. Conservation of momentum states that the momentum of car X before the collision should equal the total momentum of car X, car Y, and the cargo after the collision. Table 5-1 shows the velocity of the cars after the collision for different loads of car Y, as predicted by momentum conservation.

Just by looking at the sequence of drawings, we can get some idea of what occurs as car X interacts with an increasingly more massive car Y. As the total mass of car Y (mass of car Y + mass of the cargo) increases, the velocity of the two cars after the collision gets progressively smaller. By (e), the velocity of the two cars is so small that it cannot be noticed on the scale used for the drawings. If you compare the column labeled "Mass Y" with the column labeled "Velocity After" in Table 5-1, you see the same result. As the mass of car Y increases, the velocity of the system after the collision decreases. By the time the mass of car Y is 20 times that of car X, their velocity after the collision becomes too slow to notice.

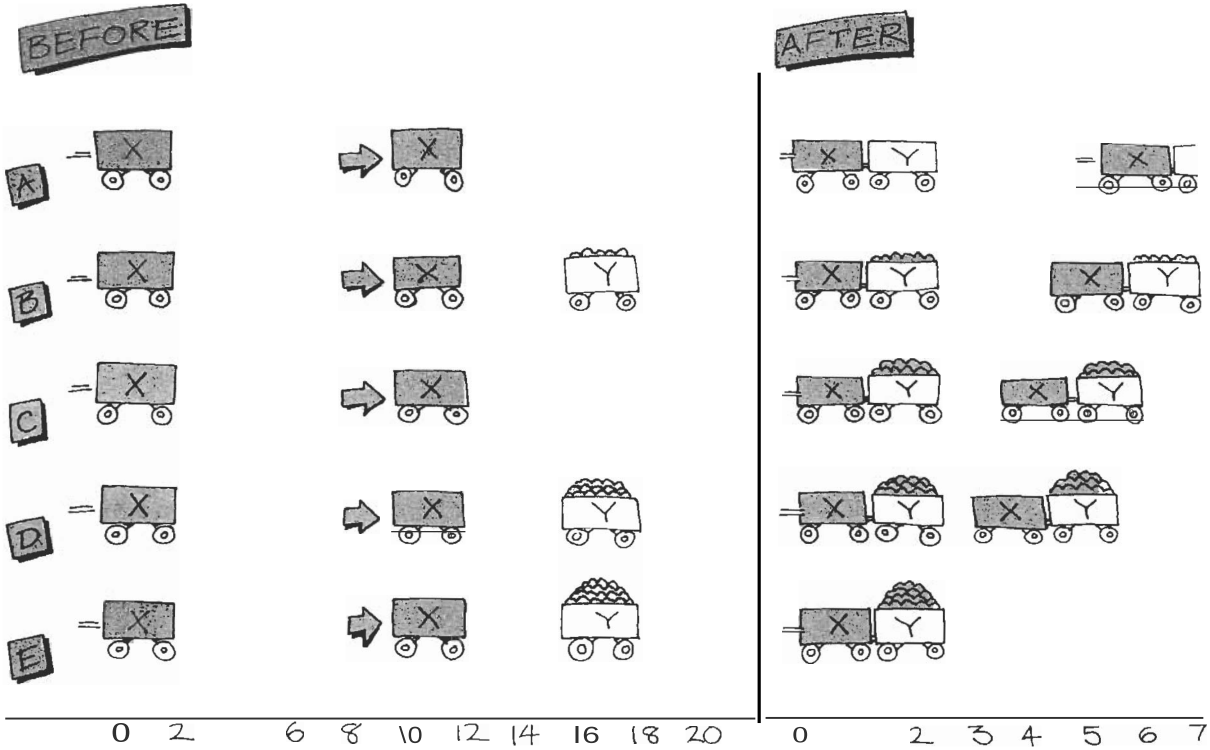


Figure 5-10 As the mass of car Y increases, the magnitude of the velocity after the interaction decreases. When the mass is extremely large, the motion may be so small that we do not perceive it.

Table 5-1 Interacting with Large Masses

Mass X	Velocity X	Momentum Before/After	Mass Y	Velocity After
1,000 kg	10 mis, right	10,000 kg . mis, right	1,000 kg	5 mis, right
1,000 kg	10 mis, right	10,000 kg . mis, right	1,500 kg	4 mis, right
1,000 kg	10 mis, right	10,000 kg . mis, right	2,333 kg	3 mis, right
1,000 kg	10 mis, right	10,000 kg . mis, right	3,000 kg	2.5 mis, right
1,000 kg	10 mis, right	10,000 kg . mis, right	19,000 kg	0.5 mis, right

Tennis Balls, Eggs, and Automobiles

Physicists think that momentum is conserved in all interactions, providing the system is closed. When objects of vastly different masses interact, motion after the interaction may be too small to detect, but it is there. When you catch a tennis ball, the ball interacts with you and you, in turn, interact with the earth. The closed system includes you, the ball, and the earth. The mass of the tennis ball is so small compared with the total mass of the earth and you that we simply do not see any motion once the ball is caught. If you remove one of these masses, such as the earth, then the objects that interact become more comparable and you begin to see motion. For example, your interaction with the earth is minimized when you stand on ice. Try catching a fastball while standing on ice and notice your motion afterwards!

When the egg interacts with the floor, the floor is attached to the building, the building is attached to the ground, and the ground is part of the earth. The mass of the egg is negligible compared to the mass of the earth. The same is true when a car collides with a bridge. Interactions with massive objects result in motion too slow to be detected. Momentum is still conserved.

Steering a spaceship is like stepping out of a boat. Dropping an egg is like running into a brick wall. Continents collide just as cars do. Air molecules bounce around, exchanging momenta in much the same fashion that billiard balls interact on a billiard table. Conservation of momentum provides a powerful tool with which to understand a number of incredibly diverse interactions. In some cases, however, convenience dictates a change in perspective. When we look at motion from the perspective of just one of the objects, we must introduce the concept of force. Chapters 6, 7, and 8 consider motion from this point of view.

CHAPTER SUMMARY

Physicists describe change in terms of interaction. When one object influences another, we say that the two objects *interact* with one another. Measurable *change* provides the *evidence for interaction*. This chapter examines interactions in which a change in velocity occurs.

Two variables, mass and velocity, help describe many of the different interactions we observe. *Mass* is a measure of the amount of matter in an object. The unit of mass is the kilogram. Unlike weight, mass does not depend on the interaction of an object with a planet; mass is the same everywhere. Mass and velocity are combined in our perceptions of interactions. We use the concept of momentum to describe this combination. *Momentum*, defined as the product of mass and velocity, is a vector quantity whose direction is the same as the direction of the velocity.

When two (or more) objects interact only with each other, their combined momentum is the same before and after the interaction. We say that *momentum* is *conserved* in interactions within a closed system. A *closed system* is a group of objects that interact with each other but not with objects outside the system. Our experience has led us to expect certain quantities to be conserved-that is, to remain the same. We call this expectation *conservation logic*. Momentum is one of the quantities identified through our use of conservation logic.

Interactions between objects of about the same mass are easy to perceive in terms of momentum conservation. When one of the objects is much larger than the other, however, the momentum of the system *seems* to disappear. Momentum is still conserved, but the motion of the larger object is too small to measure after the collision.

ANSWERS TO SELF-CHECKS

SA. a. The velocity of the object decreases.
b. The velocity of the object changes because of a change in direction.

58. a. mass
b. velocity
c. velocity

5C. Glass ball: 0.1 kg . *mis*, E

5D. Friend's momentum before interaction = 140 kg . *mis*, E. Total momentum after interaction = 140 kg . *mis*, E. You and your friend slide eastward at a velocity less than 2 *m/s*. The magnitude of the velocity depends on your mass.

5E. (0.02 kg)(300 *mis*, N) = -(10 kg)(v); v = 0.6 *mis*, S

PROBLEMS AND QUESTIONS

A. Review of Chapter Material

A1. Define each of the following terms:
Interaction
Mass
Momentum
System
Closed system
Conservation principle

A2. How is change related to interaction?

A3. In what units is momentum measured?

A4. If you know the mass and velocity of an object, how do you determine both the magnitude and direction of its momentum?

AS. Why is momentum a useful concept in describing interactions which involve a change in velocity?

A6. Under what conditions are conservation principles valid?

A7. How does our everyday use of the term to *conserve* differ from the way it is used by physicists?

A8. A system consists of two objects. Suppose that you know the mass and velocity of each object. Describe how you would determine the total momentum of the system.

A9. In a closed system, how does the momentum before the interaction compare with the momentum after the interaction?

A10. A closed system consists of two objects. During an interaction, the objects collide and stick together. Suppose you know the momentum of each object before the interaction. Describe how you would find the momentum of each object after the interaction.

A11. Why is momentum conservation difficult to observe when one of the objects is much more massive than the other?

8. Using the Chapter Material

81. Two identical bowling balls are sitting motionless on the floor. Each ball is struck

by a sledge hammer. After the interaction, ball A is moving more rapidly than ball B. If the hammers have equal masses, which one was moving more rapidly before the interaction? If they had equal speeds before the interactions, which one was more massive? Explain your answers.

B2. What is the momentum of a 70 kg sprinter moving at a velocity of 8 mis, south?

B3. A 10,000 kg railcar is coasting on level ground at 5 mis, west, when 1000 kg of snow falls vertically into it. What is the horizontal velocity of the car-snow system after the interaction?

B4. One quantity studied in nuclear interactions is parity. Before an interaction, parity is - 1; after, it is +1. Physicists studying the interaction can draw one of two conclusions. What are they?

B5. An old circus trick is firing a person from a cannon. A 75 kg circus performer is fired north at 10 mls from a 750 kg cannon. What is the velocity of the cannon after it is fired?

B6. A system consists of two ice skaters. The mass of each skater is 60 kg. Skater A is traveling west at 5 mls; skater B, east at 4 m/s. What is the total momentum of the system?

B7. A 1 kg steel ball moving at 2 mis, left, hits an identical ball that is not moving head-on. What is the speed of each ball after the interaction if:
a. They do not stick together.
b. They do stick together.

B8. Two people, one with a mass of 150 kg and the other with a mass of 75 kg, are walking toward you at identical speeds. They are so deeply involved in a conversation that they do not see you. A collision is inevitable. With which one would you choose to collide?

B9. The Army asks you to test a new cannon which has a mass of 50 kg and shoots 50-kg shells. Will you stand behind the cannon and pull the trigger?

BlO. You throw a 0.25 kg snowball at a tree with a velocity of 2 mis, SE. The tree is rigidly attached to the earth. What is the closed system? Why do you not notice the tree move as a result of the interaction?

B11. Admirals of 200 years ago fought battles by shooting broadsides-the simultaneous shooting of all cannons on one side of a ship-at their enemies. Using interaction and momentum conservation, explain why these cannons were usually mounted on wheels rather than rigidly attached to the ship.

C. Extensions to New Situations

Cl. Collisions involving insects and cars occur frequently. Usually the driver notices the collision only when washing the bugs off. The insects, by that time, are no longer. As an example of this collision, consider a 1000 kg car moving east at 25 mls and a 0.001 kg bug moving west at 1 m/s.
a. What is the total momentum of the bug-car system before the collision?
b. What must be the total momentum of the bug-car system after the collision?
c. Why does the driver of the car seldom notice the collision?

C2. Large cylinders of highly compressed gas are used to carbonate soft drinks. If a valve breaks on one of these cylinders, the gas escapes rapidly; the cylinder acts like a rocket. Such an accident happened at an Indianapolis sports arena in the early 1960s. A cylinder valve broke, the cylinder was pushed over, and it exploded, killing several people. The preliminary report on this tragedy stated that the cylinder moved because the escaping air pushed on a nearby wall. Use conservation of momentum to argue that the cylinder would move even if the wall were not there. (Safety laws now require that these cylinders be chained to the wall.)

C3. Inside a box is a marble that is free to move. You cannot see into the box, but you have been asked to describe the location and velocity of the marble in it. You can, however, roll other marbles into the box and watch them when they come back out.
a. How could rolling marbles into the box help you locate the marble in the box?
b. What would happen to each marble when the two collided?
c. How would the answer to (b) increase

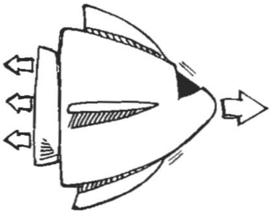
the difficulty of knowing the location of the marble in the box?

C4. A rocket is propelled by high-speed gases, which shoot out the back of the rocket (Figure 5-C4). In most rockets the amount of exhausted fuel each second is about the same throughout the flight of the rocket. About 75% of the total mass of the rocket is its fuel.

a. How does the total mass of the rocket near burnout of the fuel compare with the total mass at the beginning of the flight?

b. How will the momentum of the rocket change each second if the momentum of the exhausted fuel each second is 2500 kg . *mis,* N?

c. When will the change in speed of the rocket be greater: near the beginning of the flight or near the burnout of the fuel?



C5. On television a police officer will frequently shoot at a fleeing villain by holding the pistol in one hand. Actually, a real police officer seldom fires a weapon unless the officer has firmly gripped it with two hands. A major difference between stage guns and real guns is that the stage gun shoots a very small piece of paper.ather than a lead slug. Does this explain the difference in the way real and stage pistols are held?

C6. When you walk, you interact with the earth in order to move forward. You change your speed. Determine how momentum is conserved in this situation.

a. What objects are in the system for which momentum is conserved?

b. When you are standing still, what is the total momentum of the system? (State the momentum relative to the earth.)

C7. In this chapter, we did not discuss what happens to momentum as we change reference frames. This problem allows you to fill this gap. Suppose a billiard ball with a mass of 0.5 kg is moving left relative to the earth at 2 *m/s.* It strikes an identical second billiard ball, B, which is stationary.

a. What is the velocity relative to the earth of each ball after the interaction?

b. A second observer is moving with a velocity of 3 *mis,* right, before and after the interaction. In this reference frame, what are the velocity and momentum of each ball before the interaction?

c. What is the total momentum of the system containing the two balls before the interaction?

d. What are the velocity and momentum of each ball after the interaction?

e. What is the total momentum of the system after the interaction?

f. Use your answers to (b)-(e) to argue that momentum is conserved in the moving system.

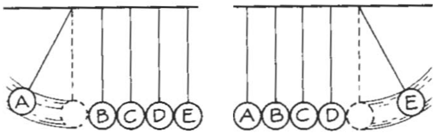
C8. A popular toy, sometimes called clackers, is shown in Figure 5-C8. Five identical balls are suspended by strings. If you hold out one ball and release it, it falls and strikes the remaining balls. Eventually the ball at the far right moves outward.

a. Use momentum conservation to predict the velocity of ball E compared to the velocity of ball A.

b. If you pull balls A and B back and release them, balls D and E move outward with the same velocity that A and B had. Would momentum still be conserved if ball E had moved outward with twice the velocity of balls A and B?

c. List other possibilities like (b) in which the momentum of the system is still conserved. (We investigate in Chapter 9 why these other possibilities do not occur.)

d. Does momentum conservation by itself uniquely determine the outcome of interactions with the steel balls?



C9. When we drop a rock it is pulled downward by an interaction with the earth called gravity. The rock's speed increases continually as it moves toward the earth. Thus the rock's momentum is constantly changing. To determine how momentum is conserved in this situation, answer the questions below.

a. What objects are included in the system for which momentum is conserved?
b. What is the total momentum of the system as the rock is released?
c. What must be the total momentum of the system as the rock starts falling?
d. In order for momentum to be conserved, how must the earth move once the rock starts failing?
e. Why do we not notice the motion of the earth?
f. What would happen if everyone in the world dropped rocks at the same time?

D. Activities

01. Watch a movie or play. Describe how fight scenes include or ignore momentum conservation. Then develop a series of stage directions for a realistic-looking fight. Describe how momentum conservation is important in your directions.

02. Momentum conservation is evident in contact sports such as hockey, rugby, soccer, or football. Watch a game and describe some plays where you saw evidence of momentum conservation.

03. Get on roller skates, ice skates, or a skateboard. Try to move about by throwing things of different masses. Describe the results and explain them in terms of momentum conservation.